Semiconductor and Quantum Physics

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Complete Course Notes

Units 1–4

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Unit 2: Semiconductor Devices

Unit 3: Quantum Physics

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Unit 1: Introduction to Solids

Comprehensive Lecture Notes

Syllabus Coverage: Energy band in solids, Fermi Dirac distribution function and Fermi level, classification: metals, semiconductors, and insulators. Intrinsic and extrinsic semiconductors, concept of holes, carrier concentration. Superconductors: temperature dependence of resistivity and critical field in superconducting materials, Meissner effect.

1.1 Formation of Energy Bands in Solids

The formation of energy bands in solids arises from the overlap of atomic orbitals when atoms come together to form a solid. Here's how it occurs:

1. **Atomic Orbitals:** In isolated atoms, electrons occupy discrete energy levels corresponding to their atomic orbitals.
2. **Close Packing:** When atoms are brought close together, their atomic orbitals begin to overlap. This overlap causes the discrete energy levels to split into many closely spaced energy levels.
3. **Band Formation:** As the number of atoms increases, the energy levels form continuous bands:
    - The overlapping atomic orbitals combine to create a range of energy levels.
    - Each energy band can accommodate a certain number of electrons, determined by the Pauli exclusion principle.
4. **Valence and Conduction Bands:** The lower energy band formed from the bonding orbitals is called the valence band, while the higher energy band, consisting of the antibonding orbitals, is known as the conduction band. The presence of a band gap between these two bands arises due to the energy difference between the filled (valence) and empty (conduction) states.

Consider a neutral sodium atom; the ground state configuration of an isolated Na atom is $1s^2 2s^2 2p^6 3s^1$. The $1s^2, 2s^2, 2p^6$ levels are completely filled while the $3s^1$ is half filled, each occupying a specific energy level. The energy levels of sodium become bands when the atoms lie close together. For large atomic distances, the interactions between atoms are negligible and the energy levels remain unsplit. With the decrease in interatomic distance, the $3s$ level splits into bands and with further decrease in distance, the $2p$ level also starts to split. The $1s$ and $2s$ levels do not split at all.

If there are N atoms in a solid, there are N allowed states in each band. Each state can be occupied by a maximum of two electrons with opposite spin. Thus, each band can be occupied by $2N$ electrons. When the atoms are part of a solid, they interact with each other, and the electrons have slightly different energies.

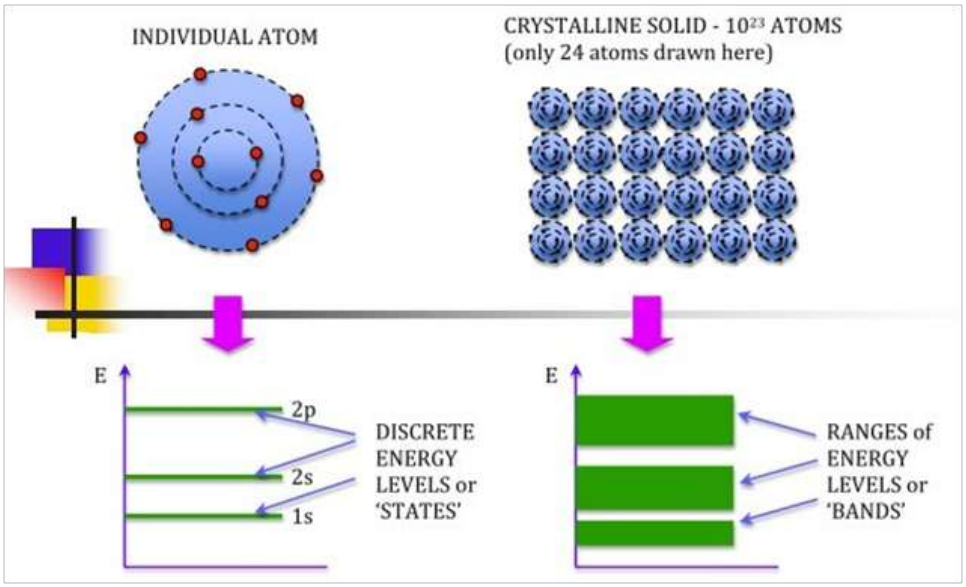


Figure 1: Formation of energy band

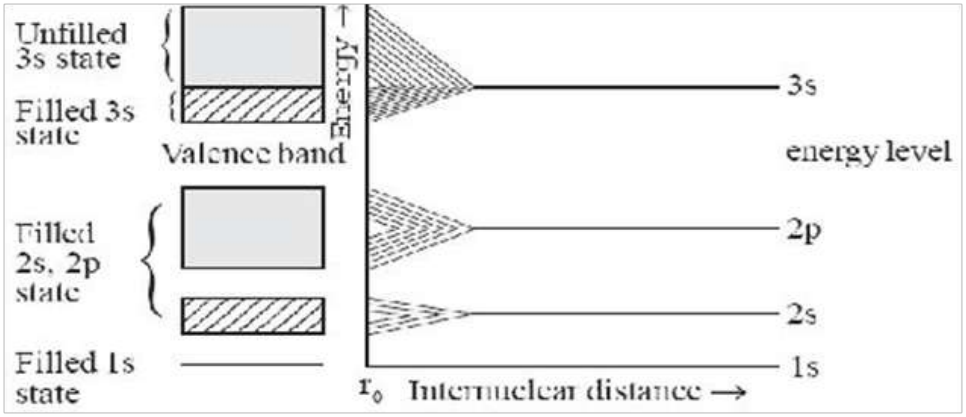


Figure 2: Energy bands in Sodium.

1.2 Conduction Band, Valence Band, and Forbidden Energy Gap in Solids

1. **Valence Band:**
   - The valence band is the energy band that contains the electrons responsible for chemical bonding and electrical conductivity in solids. It is filled with electrons that are involved in bonding between atoms.
   - The top of the valence band is the highest energy level that is occupied by electrons at absolute zero temperature.
2. **Conduction Band:**
   - The conduction band is the energy band above the valence band. It contains the energy levels that electrons can occupy when they gain enough energy to break free from the bonds holding them in the valence band.
   - Electrons in the conduction band are free to move throughout the material, contributing to electrical conductivity.

3. **Forbidden Energy Gap (Band Gap):**
    - The forbidden energy gap (or band gap) is the energy difference between the valence band and the conduction band. It represents the energy required for an electron to transition from the valence band to the conduction band.
    - The size of the band gap determines the electrical and optical properties of a material:
        - **Conductors:** No band gap, as the conduction and valence bands overlap (e.g., metals).
        - **Semiconductors:** Small band gap (typically less than 3 eV), allowing for some electron excitation at room temperature (e.g., silicon).
        - **Insulators:** Large band gap (greater than 3 eV), making it difficult for electrons to jump from the valence band to the conduction band (e.g., glass).

1.3 Classification of Metals, Semiconductors, and Insulators

We can classify materials into 3 categories based on the band gap between the conduction and valence band:

1. **Metal:** If the conduction and valence bands overlap, electrons are free to roam inside the material due to the overlapping of the valence and conduction bands. Electrons don't need extra energy to jump into the conduction band.
2. **Semiconductor:** If there exists some energy difference (< 3 eV) between the valence and conduction band, in that case, electrons need excitation energy (< 3 eV) to be in a state of free conduction (i.e. to jump into the conduction band).
3. **Insulators:** If there exists a large energy difference between the valence and conduction band (> 3 eV), materials are called insulators. Electrons need very large energy to be in the state of free roaming through materials.

Note: The energy difference between valence and conduction bands is always measured from the valence band maxima to the conduction band minima.

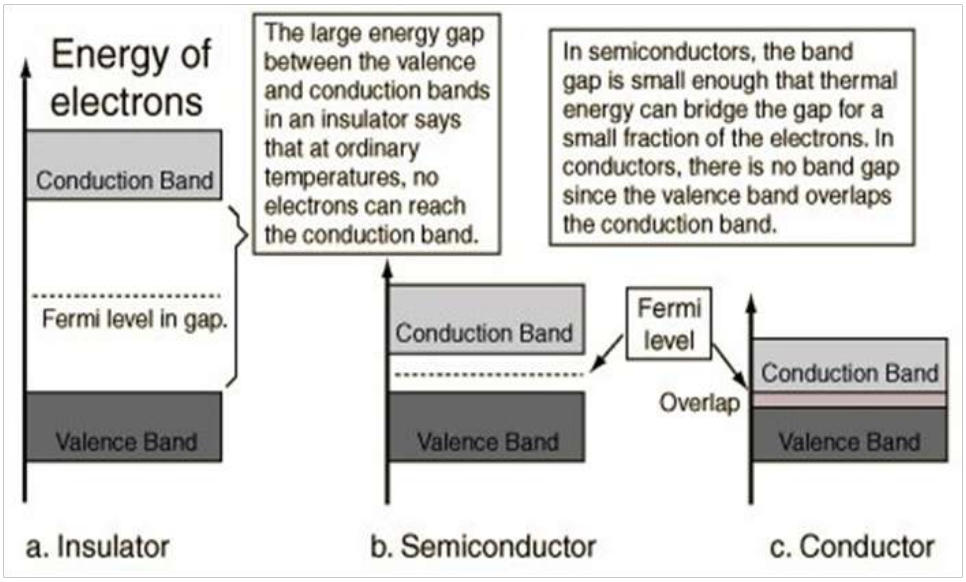


Figure 3: Energy band in insulator, semiconductor and conductor.

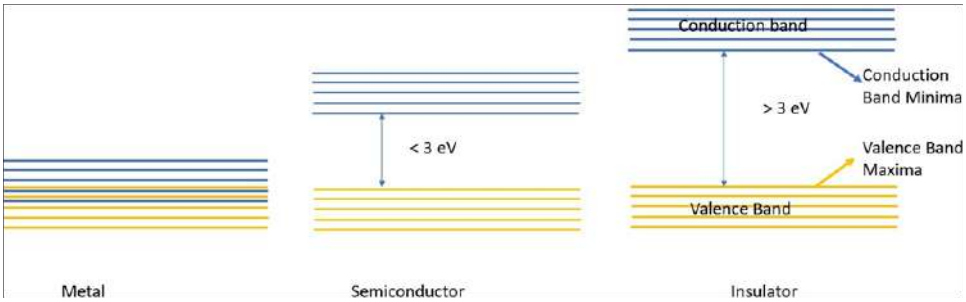


Figure 4: Classification of metal,semiconductor,and insulators based on the energy difference between valence and conduction band.

1.4 Types of Semiconductor

There are two types of semiconductors that contrast in behavior, structure, and performance: **Intrinsic Semiconductors** and **Extrinsic Semiconductors**. The core difference is that intrinsic semiconductors are completely pure materials, whereas extrinsic semiconductors are chemically altered with added impurities to boost their electrical conductivity.

1.4.1 Intrinsic Semiconductors

Intrinsic semiconductors are composed of a single, highly refined element.

- **Atomic Structure:** Elements like Silicon (Si) and Germanium (Ge) have 4 valence electrons. They form rigid covalent bonds with four surrounding atoms.
- **Conduction Mechanism:** At absolute zero (0 K), all electrons are locked in these covalent bonds. The material acts as an insulator. As temperature increases, thermal energy breaks a few covalent bonds. This liberates free electrons into the conduction band, leaving behind vacant spaces called holes in the valence band.
- **Formula:** The intrinsic carrier concentration n_i is defined by:

$$n_e = n_h = n_i \tag{1}$$

  Where n_e is electron density and n_h is hole density.

1.4.2 Extrinsic Semiconductors

The electrical properties of pure semiconductors are modified by the addition of certain impurities. Because intrinsic conductivity is too weak for real-world devices, materials undergo a process called doping. This introduces microscopic amounts of foreign atoms (roughly 1 per million host atoms) to amplify conductivity exponentially. A semiconductor in which the charge carriers originate due to impurity atoms added to it is called an extrinsic semiconductor. Extrinsic semiconductors are split into two groups based on the chemical dopant chosen:

1.4.2.1 N-Type Semiconductor (Negative)

When a small concentration of pentavalent impurities (like antimony, arsenic, phosphorus etc.) is added to a pure semiconductor, four electrons are bonded covalently with four neighbouring pure semiconductor atoms. The fifth electron is loosely bound to the parent atom. Even at room temperature, this fifth electron becomes free to move without adding any new hole. Clearly, the pentavalent elements donate negative charges (i.e., electrons) and hence such impurities are

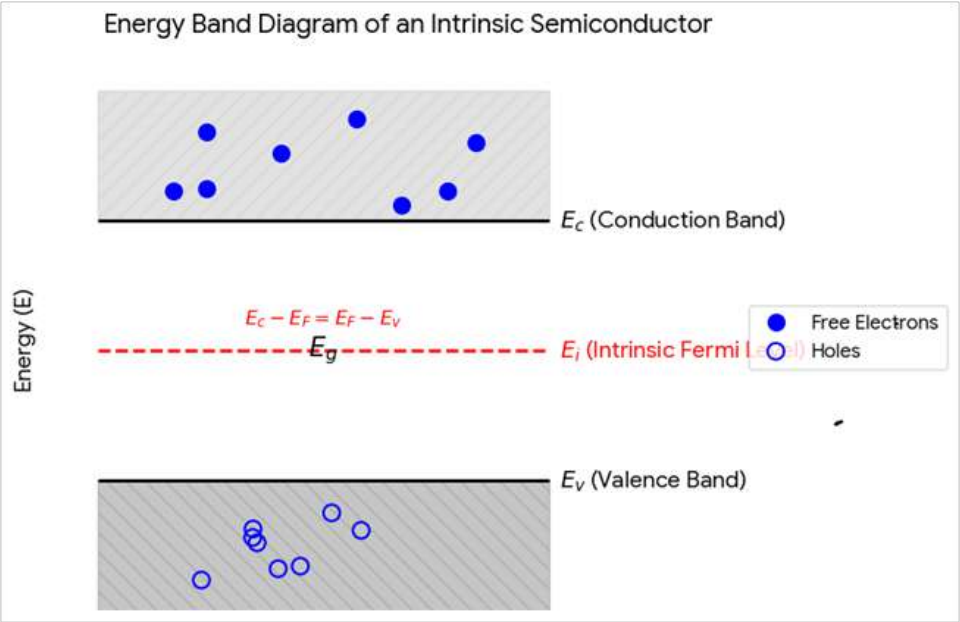


Figure 5: Energy band in intrinsic semiconductor.

called **donor impurities** or n-type impurities. The semiconductors doped with pentavalent impurities are called n-type semiconductors.

These impurities introduce new energy states slightly below the bottom of the conduction band. Here, E_d represents the energy level corresponding to the fifth electron of donor impurities and is called the **donor level**. At absolute zero, donor levels are occupied by the donated electrons. But as the temperature rises, these electrons are excited into the conduction band and become the majority charge carriers. If the temperature is sufficiently high, electron-hole pairs are also generated due to breaking of covalent bonds. Therefore, **electrons are majority carriers** in n-type semiconductors.

- **Dopant:** Pentavalent elements with 5 valence electrons, such as Phosphorus (P), Arsenic (As), or Antimony (Sb).
- **Mechanism:** Four of the dopant's valence electrons bond into the silicon lattice. The fifth electron stays loosely bound and easily detaches at room temperature to become a free charge carrier.
- **Carrier Dominance:** Electrons are the majority carriers, while holes generated by ambient heat are the minority carriers ($n_e \gg n_h$).

1.4.2.2 P-Type Semiconductor (Positive)

When a small concentration of trivalent impurities (like aluminium, gallium, indium etc.) is added to a pure semiconductor, three electrons of the added impurity are bonded covalently with three neighbouring pure semiconductor atoms. The impurity atom needs one more electron to complete its bond which is supplied by the semiconductor atom. In this process, a vacant electron site or hole is generated. Clearly, such impurities accept electrons and hence are called **acceptor impurities**. The semiconductor doped with trivalent impurities is called a p-type semiconductor.

These impurities introduce new energy levels slightly above the top of the valence band. Here E_a represents the energy level corresponding to acceptor impurities and are called **acceptor**

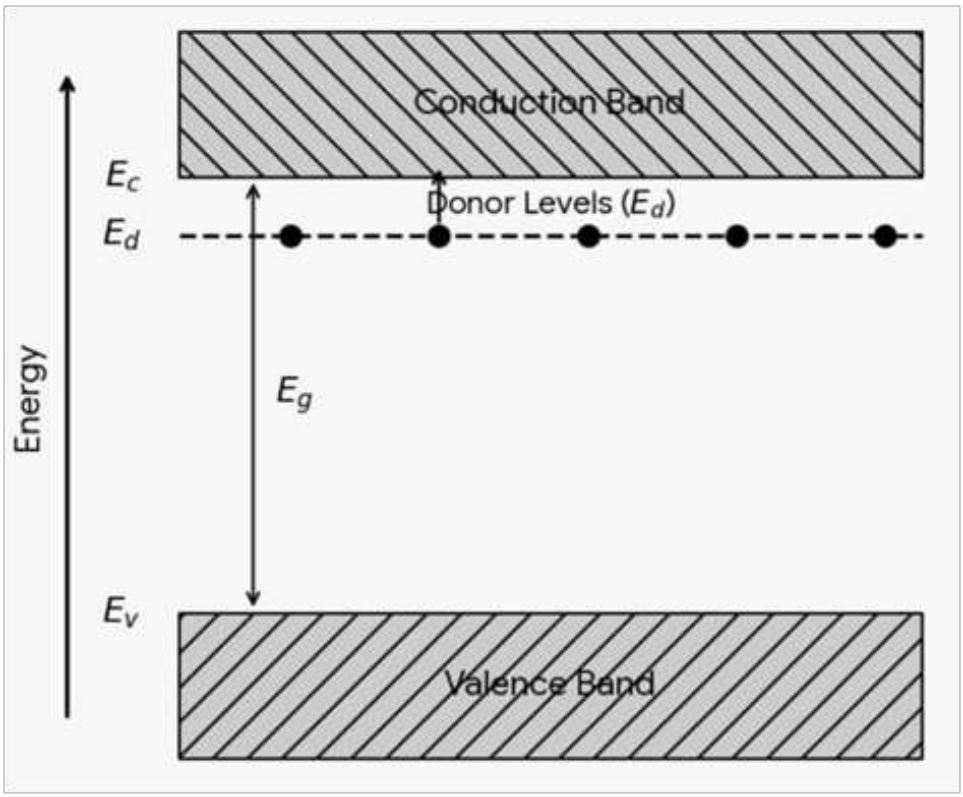


Figure 6: Energy band in n-type semiconductor.

levels. Trivalent impurities cause extra holes in the crystal. So, acceptor levels are occupied by holes at absolute zero in the vicinity of their respective ions. But as the temperature is raised, the holes in the p-type impurity are occupied by the electrons from the valence band, giving rise to holes in the valence band. The energies for the holes are higher near the valence band and decrease vertically upward in the energy level diagram. If the temperature is increased sufficiently, the electron-hole pairs are generated due to breaking of covalent bonds. Thus, in p-type semiconductors, holes are more than electrons and hence, **holes are majority carriers** and electrons are minority carriers.

- **Dopant:** Trivalent elements with 3 valence electrons, such as Boron (B), Aluminium (Al), or Indium (In).
- **Mechanism:** The dopant forms bonds with three adjacent silicon atoms. The fourth bond remains incomplete, creating a natural electronic vacancy or "hole" that eagerly accepts traveling electrons.
- **Carrier Dominance:** Holes are the majority carriers, while thermally freed electrons are the minority carriers ($n_h \gg n_e$).

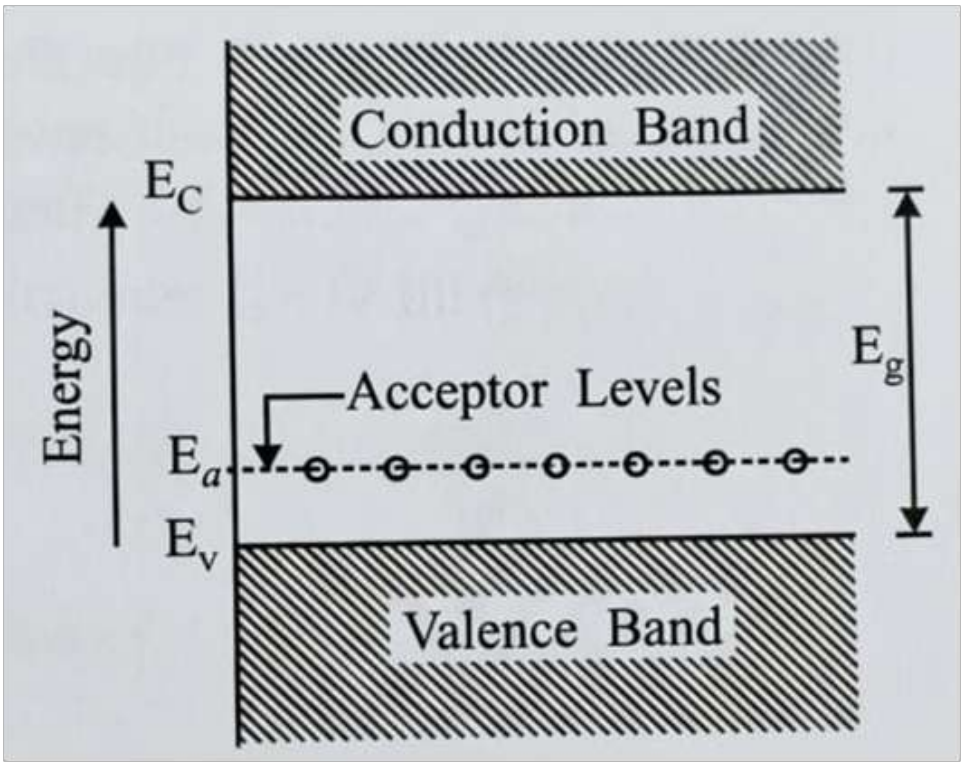


Figure 7: Energy band in p-type semiconductor.

Comparison Table

Property	**Intrinsic Semiconductor**	**Extrinsic Semiconductor**
Purity	100% pure, untampered element	Doped with external impurities
Conductivity	Extremely low at room temperature	Very high and highly controllable
Charge Carriers	Electrons equal the number of holes ($n_e = n_h$)	Electrons do not equal holes ($n_h \neq n_e$)
Control Factors	Dependent solely on temperature	Dependent on temperature and amount of dopants
Classification	None	Separated into N-type and P-type
Fermi Level	Exactly in the middle.	Shifts near the band
Examples	Pure Silicon (Si), Germanium (Ge)	Silicon doped with Phosphorus or Boron

Table 1: Comparison between Intrinsic and Extrinsic Semiconductors

1.5 What Is a Hole?

Silicon (Si) and Germanium (Ge) crystallize in a diamond lattice structure, where every atom is surrounded by four equidistant nearest neighbours at the corners of a tetrahedron. Both belong to Group IV of the periodic table and possess four valence electrons. Each atom forms four covalent bonds with its four nearest neighbours by sharing valence electrons of opposite spin.

At absolute zero (0 K), every valence electron is locked into a covalent bond, leaving no free carriers — the crystal behaves as a perfect insulator. At room temperature (∼300 K), however,

some electrons acquire enough thermal (kinetic) energy to break free of their covalent bonds. The minimum energy needed to break a bond is about 0.72 eV for Ge and 1.1 eV for Si.

When an electron escapes a covalent bond, it leaves behind an incomplete bond — a vacancy. This vacancy is what we call a **hole**.

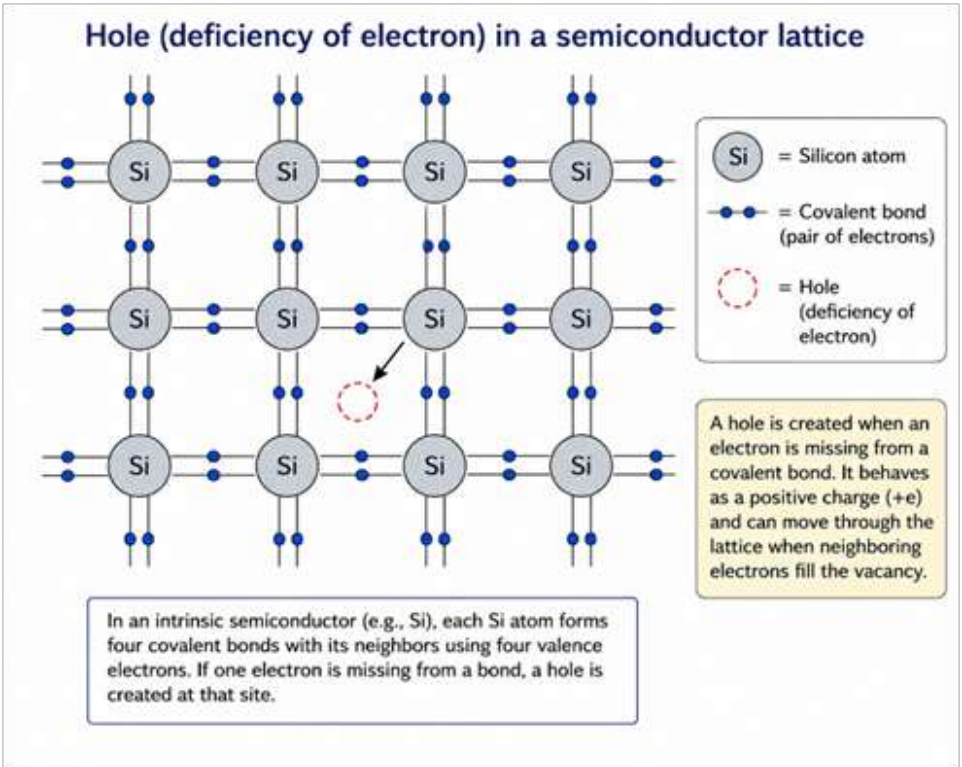


Figure 8: Formation of hole.

1.5.1 Hole as a Positive Charge Carrier

A hole may be imagined to behave like a positively charged particle, equal and opposite in charge to that of an electron, and it can take part in the conduction of electricity. Under the action of an external electric field, an electron from a nearby filled covalent bond (having almost the same energy as the hole) may move over and fill the hole. This electron, in turn, leaves a new hole behind at its original position. The net effect is that the hole appears to move in a direction opposite to the direction of motion of the electrons filling it — even though no physical positive particle actually travels through the crystal.

Both electrons and holes end up contributing to current flow in the same direction, because their charges and directions of motion are both opposite to each other. As far as electrical conduction is concerned, a hole behaves exactly like a particle carrying a charge equal and opposite to that of an electron.

1.5.2 Electron-Hole Pair Generation and Recombination

- **Generation:** The thermal breakage of a covalent bond, which simultaneously creates one free electron and one hole, is known as electron-hole pair generation.
- **Recombination:** A free electron moving randomly through the crystal may occasionally collide with a broken covalent bond (a hole) and combine with it. The free electron is thereby converted back into a bound electron, and the electron-hole pair is lost from the conduction process. This is called recombination.

- **Equilibrium:** In thermal equilibrium, the rate of generation equals the rate of recombination, so a definite, temperature-dependent number of electron-hole pairs exists per unit volume of the crystal at any given time.

In a pure (intrinsic) semiconductor, the number of free electrons is always equal to the number of holes, since every generation event creates exactly one of each:

$$n = p = n_i \tag{2}$$

where n_i is called the intrinsic carrier concentration.

1.5.3 Holes in the Band Theory Picture

The hole concept can also be understood using the energy band picture of solids. The valence band is the outermost band that is completely filled with electrons, while the conduction band is the allowed (normally empty) band just above it, separated from the valence band by the forbidden energy gap, E_g.

When an electron in the valence band gains enough thermal energy to jump across E_g into the conduction band, it leaves behind an unoccupied state in the otherwise filled valence band. This unoccupied state — a hole — can also contribute to current flow. In this sense, a hole is considered a quasiparticle carrying a charge equal and opposite to that of an electron.

As with the covalent-bond picture, this band-to-band jump is called electron-hole pair generation, and the reverse process — an electron falling from the conduction band back into a hole in the valence band — is called recombination. The energy released during recombination, roughly equal to E_g, is either converted to heat in the crystal lattice or emitted as electromagnetic radiation (a photon), depending on whether the material is an indirect or direct band-gap semiconductor.

1.5.4 Effective Mass and the Hole

To simplify the quantum-mechanical motion of carriers in a periodic crystal field, electrons and holes are treated as classical particles possessing an effective mass (m_e^* for the electron and m_h^* for the hole) rather than their true rest mass. This effective mass depends on the curvature of the energy band in wavevector (k) space and can be larger or smaller than the free-electron rest mass.

Near the maximum of the valence band, the effective mass of an electron works out to be negative. It is precisely this negative effective mass that gives rise to the notion of a hole — mathematically, a state with negative mass and negative charge (an electron near the band maximum) behaves identically to a fictitious particle of positive mass and positive charge (a hole). This equivalence allows carriers to be analysed using ordinary Newtonian mechanics under an applied electric field.

1.5.5 Majority and Minority Carrier Roles

Although electrons and holes always appear in equal numbers in an intrinsic semiconductor, doping changes this balance:

Semiconductor Type	**Role of Holes**	**Role of Electrons**
Intrinsic (pure)	Equal to electrons ($n = p$)	Equal to holes ($n = p$)
n-type (donor-doped)	Minority carrier	Majority carrier
p-type (acceptor-doped)	Majority carrier	Minority carrier

Points to remember

- A hole is the vacancy left behind when an electron breaks free of a covalent bond, or equivalently, an unoccupied state at the top of the valence band.
- A hole behaves like a particle carrying a positive charge equal in magnitude to the electron's charge.
- Hole motion under an electric field is opposite to actual electron motion, but both contribute current in the same direction.
- Electron-hole pairs are continuously generated (thermal bond-breaking / band-to-band excitation) and recombined, reaching a dynamic equilibrium at a given temperature.
- In an intrinsic semiconductor, $n = p = n_i$; doping breaks this symmetry, making holes either the majority carrier (p-type) or minority carrier (n-type).
- The hole is mathematically justified through the concept of negative effective mass of electrons near the valence band maximum.

1.6 Carrier Concentration in Semiconductors

To calculate the conductivity of a semiconductor, the density of electrons in the conduction band and the density of holes in the valence band must first be known. This requires two ingredients: the density of allowed states at each energy, and the probability that a given state is occupied. We assume the temperature and carrier densities are low enough that the electron gas in the conduction band and the hole gas in the valence band behave as non-degenerate gases.

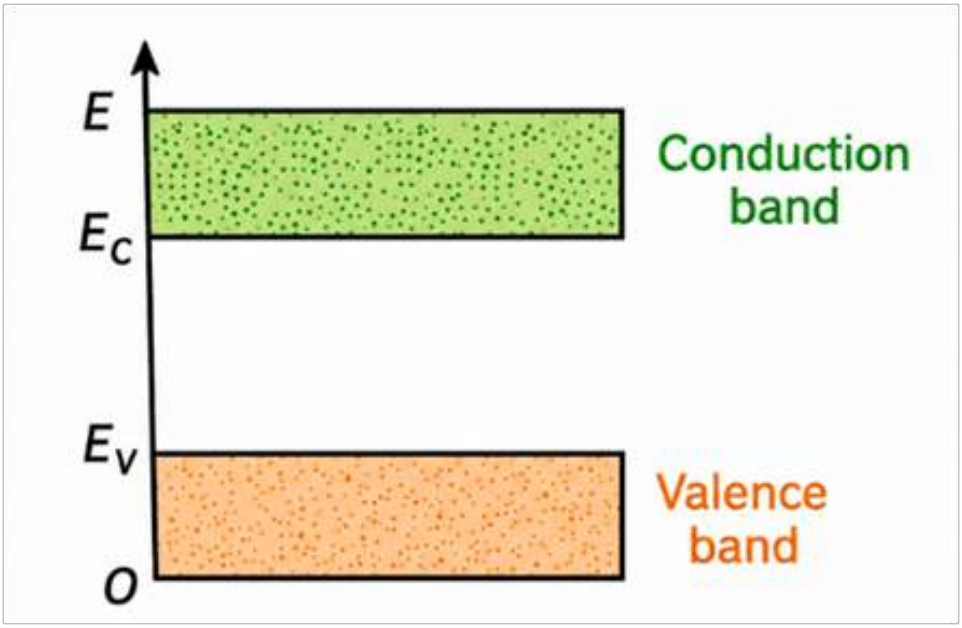


Figure 9: Valence and Conduction band.

- **Density of states, $g(E)$:** Gives the number of available quantum states per unit energy near the bottom of the conduction band:

$$g(E) = C(E - E_c)^{1/2} \tag{3}$$

Where, $C = 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2}$.

- **Fermi-Dirac distribution, $f(E)$:** Gives the probability that a state of energy E is occupied by an electron at temperature T:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \tag{4}$$

where E_F is the Fermi energy — the energy level at which the occupation probability equals 1/2.

1.6.1 Electron Concentration in the Conduction Band

The number of electrons occupying states between E and $E + dE$ per unit volume is $dn = f(E) \cdot g(E) \cdot dE$. Integrating over the whole conduction band (the upper limit may be extended to infinity since $f(E)$ becomes negligible far above E_c):

$$n = \int_{E_c}^{\infty} f(E)g(E)dE \tag{5}$$

At low temperature and for $E \geq E_c$, $(E - E_F) \gg kT$, so the Fermi function simplifies to $f(E) \approx \exp\left(-\frac{E-E_F}{kT}\right)$. Substituting this and $g(E)$ into the integral:

$$\begin{aligned} n &= C \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left(-\frac{E - E_F}{kT}\right) dE \\ &= C \exp\left(-\frac{E_c - E_F}{kT}\right) \int_0^{\infty} (kT)^{3/2} x^{1/2} e^{-x} dx \quad [\text{putting } x = \frac{E - E_c}{kT}] \\ &= C \exp\left(-\frac{E_c - E_F}{kT}\right) (kT)^{3/2} \frac{\sqrt{\pi}}{2} \\ &= 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left(-\frac{E_c - E_F}{kT}\right) \\ &= N_c \exp\left(-\frac{E_c - E_F}{kT}\right) \end{aligned} \tag{3}$$

Here $N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$ is called the **effective density of states in the conduction band**.

1.6.2 Hole Concentration in the Valence Band

Near the top of the valence band, the density of states takes the analogous form:

$$g(E) = C(E_v - E)^{1/2} \tag{6}$$

Where $C = 4\pi(2m_h^*/h^2)^{3/2}$, m_h^* is the effective mass of a hole and E_v the energy at the top of the valence band. Since a hole represents an unoccupied electron state, the relevant occupation probability is $1 - f(E)$; the probability that a state is NOT occupied by an electron. At low temperature, for $E \leq E_v$, $E_F - E \gg kT$, so $1 - f(E) \approx \exp\left(-\frac{E_F - E}{kT}\right)$. Hence the hole density in the valence band is given by:

$$\begin{aligned} p &= \int_{-\infty}^{E_v} C(E_v - E)^{1/2} \exp\left(-\frac{E_F - E}{kT}\right) dE \\ &= N_v \exp\left(-\frac{E_F - E_v}{kT}\right) \end{aligned} \tag{5}$$

Where $N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$ is called the **effective density of states in the valence band**.

For an intrinsic semiconductor, $n = p$. Equating the expressions for n and p and solving for E_F gives:

$$E_F = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_v}{N_c}\right) = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln\left(\frac{m_h^*}{m_e^*}\right) \tag{7}$$

- If $m_h^* = m_e^*$, the Fermi level lies exactly at the middle of the forbidden energy gap.
- Since in general $m_h^* > m_e^*$, the Fermi level rises slightly above midgap as temperature increases.

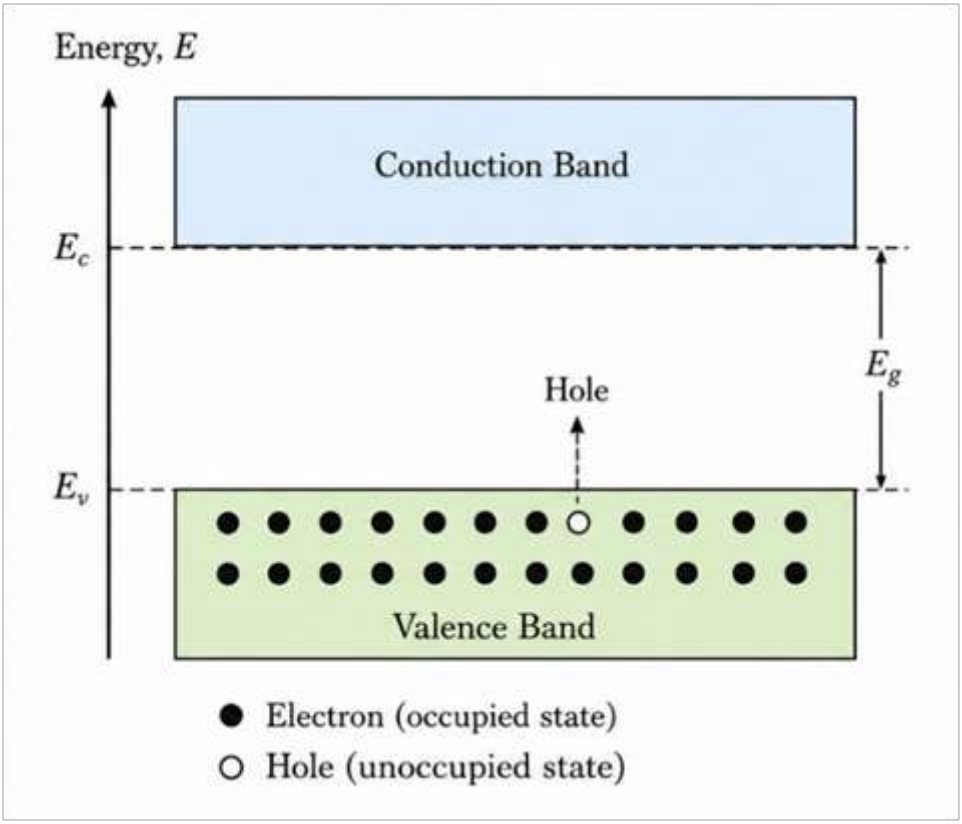


Figure 10: Hole and electron in bands.

1.6.3 Intrinsic Carrier Concentration (n_i)

Substituting the Fermi level expression back into either n or p (or simply multiplying n and p and taking the square root) gives the intrinsic carrier concentration:

$$n_i = \sqrt{N_c N_v} \cdot e^{-E_g/2kT} = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} \cdot e^{-E_g/2kT} \tag{6}$$

Where $E_g = E_c - E_v$ is the band gap. Since the band gap decreases roughly linearly with temperature, $E_g = E_{go} - \beta T$ (E_{go} = gap at 0 K, β = material constant). This confirms that the intrinsic carrier concentration is highly and exponentially dependent on temperature.

1.6.4 Carrier Concentration in Extrinsic Semiconductors

From Mass Action Law: Multiplying the general expressions for n and p together, the Fermi energy E_F cancels out entirely, leaving:

$$np = n_i^2 \tag{8}$$

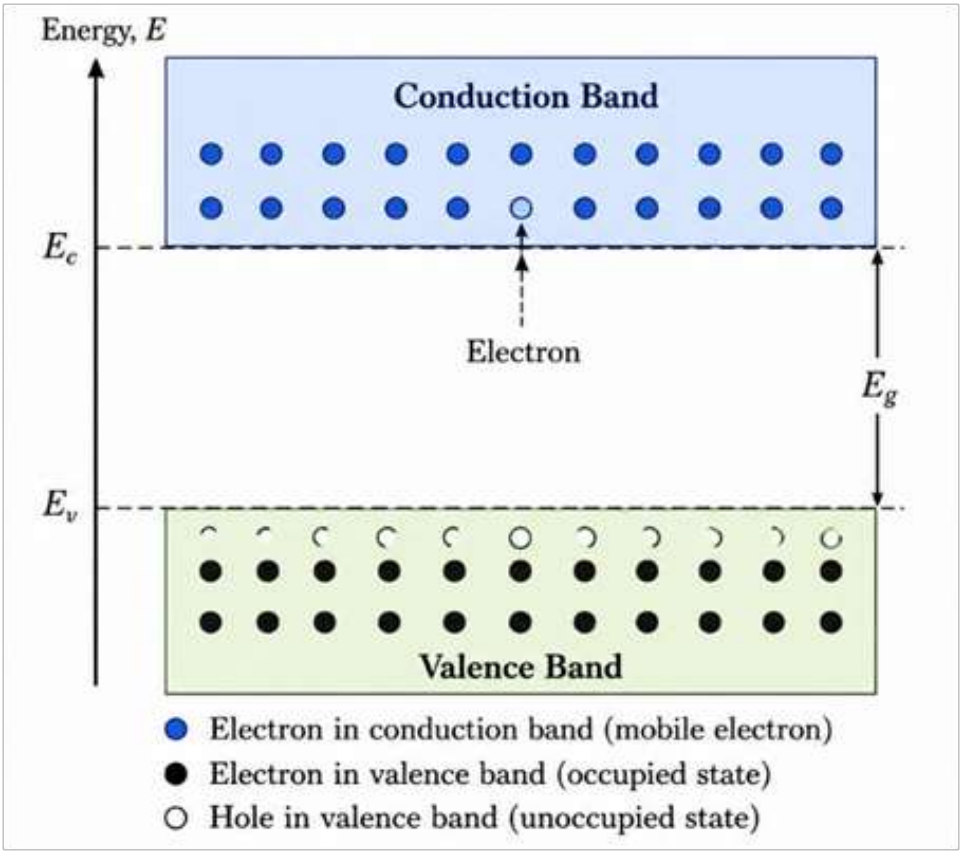


Figure 11: Hole and electron in bands.

This states that the product of the free-electron and hole concentrations is constant at a given temperature, regardless of the amount of donor or acceptor doping. Adding n-type impurities raises the electron concentration above the intrinsic level but lowers the hole concentration below it (because more electrons increase the recombination rate), and the reverse happens on adding p-type impurities.

Assuming every donor and acceptor atom is ionized, the crystal as a whole remains electrically neutral, i.e., total negative charge equals total positive charge:

$$n + N_a = p + N_d \tag{9}$$

Where N_a and N_d are the acceptor and donor impurity concentrations respectively.

- **For n-type Semiconductor ($n \gg p$):** $n \approx N_d - N_a$ and $p = n_i^2/n$
- **For p-type Semiconductor ($p \gg n$):** $p \approx N_a - N_d$ and $n = n_i^2/p$

1.7 Fermi Dirac Distribution Function

- **Definition:** The Fermi Dirac distribution function describes the probability that a fermion, such as an electron, will occupy a particular energy level at a given temperature.
- **Material Conductivity:** This function is essential in electronics for understanding how many free electrons are available to conduct electricity in a material.
- **Energy Band Theory:** The distribution function ties into energy band theory, helping to explain the concentration of electrons in the conduction band.

- **Temperature Effects:** The Fermi-Dirac distribution function shows how electron energy states change with temperature, affecting the material's conductive properties.

1.7.1 Fermi-Dirac Distribution Expression

Mathematically, the probability of finding an electron in the energy state E at the temperature T is expressed as:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} \tag{1}$$

Where:

- $k_B = 1.38 \times 10^{-23}$ J K^{-1} is the Boltzmann constant.
- T is the absolute temperature.
- E_F is the Fermi level or the Fermi energy.

Let's discuss the following cases to understand the significance of Fermi Dirac function. **At temperature $T = 0$ K:**

$$f(E) = \begin{cases} 1 & \text{if } E < E_F \\ 0 & \text{if } E > E_F \end{cases}$$

This means at 0 Kelvin all the energy states below the Fermi energy are occupied, and all energy states above E_F are unoccupied. At 0 kelvin $f(E)$ is just a step function. Also, at any temperature T, for $E = E_F$, $f(E) = 1/2$.

If we increase the temperature, some of the states above E_F will get populated. If we further increase the temperature (T_2), $f(E)$ will increase. However, irrespective of temperature, the probability at Fermi energy E_F will always be half.

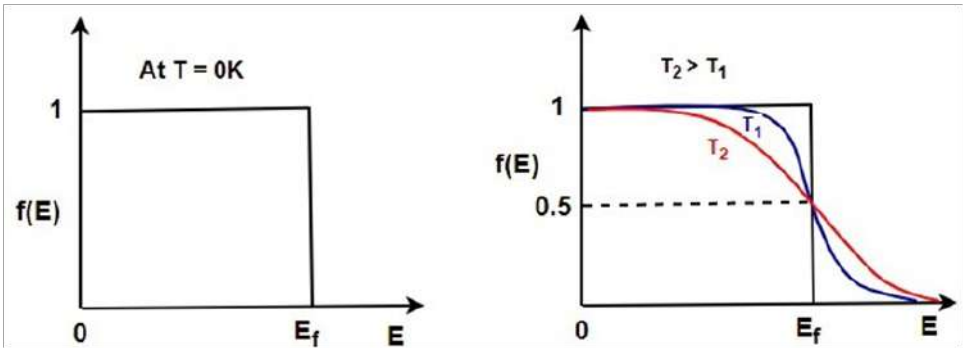


Figure 12: Fermi-Dirac distribution.

1.7.2 Fermi Level

The highest energy level that an electron can occupy at absolute zero temperature is known as the Fermi Level. For an intrinsic semiconductor, the Fermi level lies close to the middle of the forbidden gap, between the valence and conduction bands, because the electron concentration in the conduction band equals the hole concentration in the valence band (electron-hole symmetry).

1.8 Superconductors

The phenomenon of superconductivity was discovered by Kammerlingh Onnes in 1911 when he measured the resistivity of pure mercury drops at low temperature. He observed that the

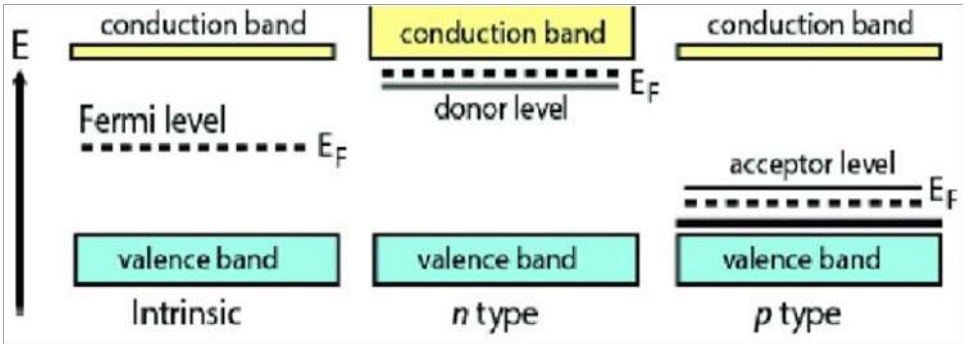


Figure 13: Fermi-levels in intrinsic, n-type and p-type semiconductor.

electrical resistivity of pure mercury drops to zero at a temperature of about 4.2 K. Kammerlingh Onnes recognized this as a new phenomenon in which mercury passes into a new state called superconductivity.

When a substance loses its electrical resistance i.e. a current can continue through it without changing its value, the phenomenon is known as **superconductivity**. Or, when the electrical resistance of a substance drops suddenly to zero when it is cooled below a certain temperature, the phenomenon is known as superconductivity. The substance showing this property is known as a superconductor. Example: Mercury, silver, lead, gallium, iridium etc.

1.8.1 Temperature dependence of resistivity in superconductors

Metals are good conductors of electricity as they have plenty of free electrons, however they offer resistance to the flow of charges. Even at 0 K, the metals offer some resistance called residual resistance.

On the contrary, the resistance of superconductors in a non-superconducting state decreases with temperature as in the case of normal metals. But at a particular temperature T_c, the resistivity abruptly drops to zero. The temperature at which a normal material turns into a superconductor is known as the **critical temperature**.

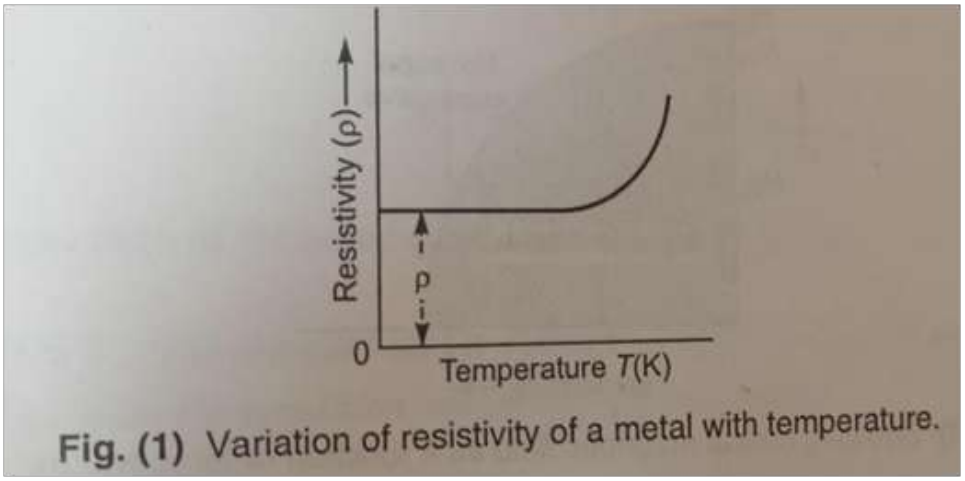


Figure 14: Resistivity of metal with temperature.

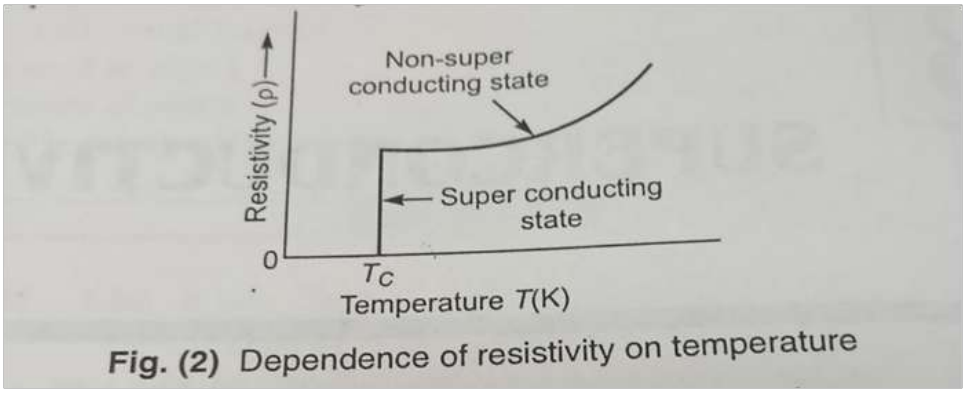


Figure 15: Resistivity of superconductor with temperature.

Table 2: Properties of some selected superconductors in chronological order

Year	T_c **(K)**	**Material**	**Class**	**Crystal Structure**	**Type**	H_c^* **(MA m^{-1})**
1911	4.2	Hg	Metal	Tetragonal	I	0.033
1913	6.2	Pb	Metal	fcc	I	0.064
1930	9.25	Nb	Metal	bcc	II	0.164
1940	15	NbN	Interstitial compound	NaCl	II	12.2
1950	17	V_3Si	Intermetallic compound	β-tungsten (W_3O)	II	12.4
1954	18	Nb_3Sn	Intermetallic compound	W_3O	II	18.5
1960	10	Nb-Ti	Alloy	bcc	II	11.9
1964	0.7	$SrTiO_3$	Ceramic	Perovskite	II	Small
1970	20.7	Nb_3(Al, Ge)	Intermetallic compound	W_3O	II	34.0
1977	23	Nb_3Ge	Intermetallic compound	W_3O	II	29.6
1986	34	$La_{1.85}Ba_{0.15}CuO_4$	Ceramic	Tetragonal	II	43
1987	90	$YBa_2Cu_3O_7$	Ceramic	Orthorhombic	II	111
1988	108	Bi cuprates	Ceramic	Orthorhombic	II	–
1988	125	Tl cuprates	Ceramic	Orthorhombic	II	–

*extrapolated to 0 K. H_{c2} for type II materials. (1 Oersted = 79.6 A m^{-1})

1.8.2 Effect of External Field

In 1913 Kammerlingh Onnes observed that superconductivity is destroyed if a sufficiently strong magnetic field is applied. The superconducting material restores its normal resistance when a strong magnetic field is applied. The minimum magnetic field which is necessary to regain the normal resistivity is called the **critical field (H_c)**. If the applied magnetic field exceeds the critical value $H_c(0)$, the superconducting state is destroyed.

The dependence of the critical field upon the temperature is given by:

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right] \tag{10}$$

Where $H_c(T)$ is the critical field at T temperature, $H_c(0)$ is the critical field at absolute zero

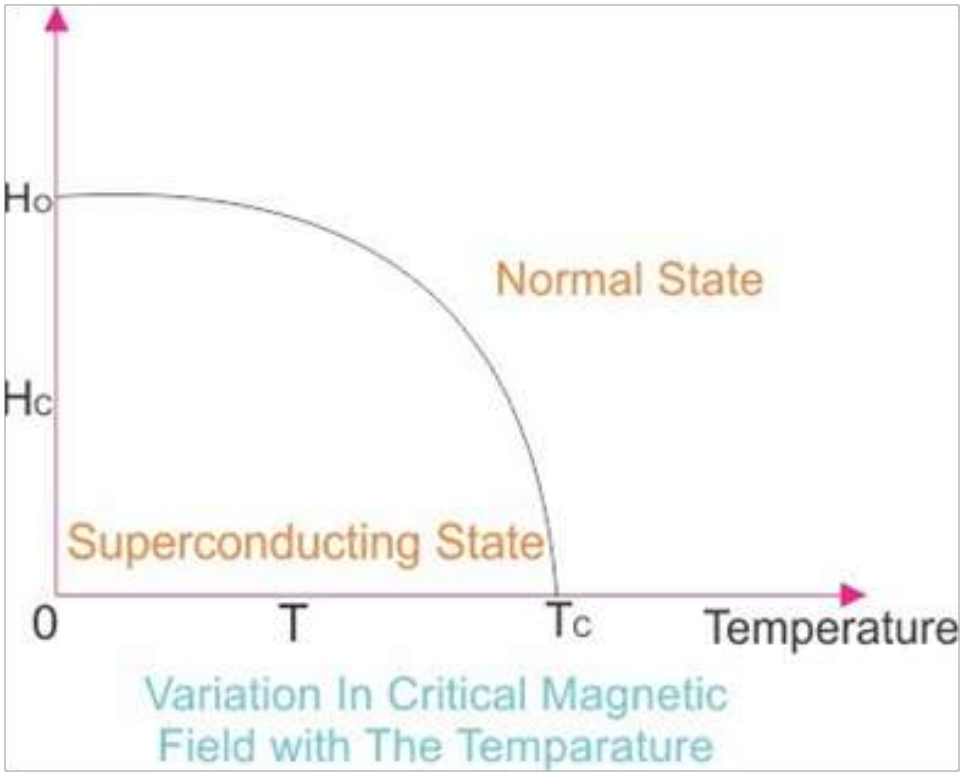


Figure 16: Variation of critical magnetic field with temperature.

(0 K) temperature, and T_c is the critical temperature or transition temperature.

1.8.3 Meissner Effect

When a material transitions from the normal to the superconducting state, it actively expels magnetic fields from its interior—a phenomenon known as the **Meissner effect**. This exclusion of magnetic fields is fundamentally different from the perfect diamagnetism that would arise solely due to zero electrical resistance.

In a perfect conductor (with zero resistance), an imposed magnetic field would generate persistent current loops to cancel the applied field, as dictated by Lenz's law. However, if the material initially possessed a steady magnetic field before being cooled below its superconducting transition temperature, the field would remain unchanged because no electromotive force (EMF) would be induced (per Faraday's law) to drive currents. This suggests that the Meissner effect is a unique and active property of superconductors, separate from their zero-resistance behaviour.

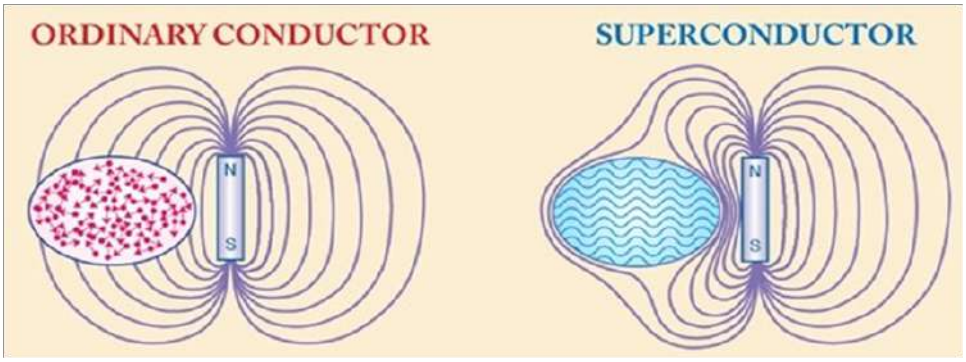


Figure 17: Perfect conductor vs superconductor.

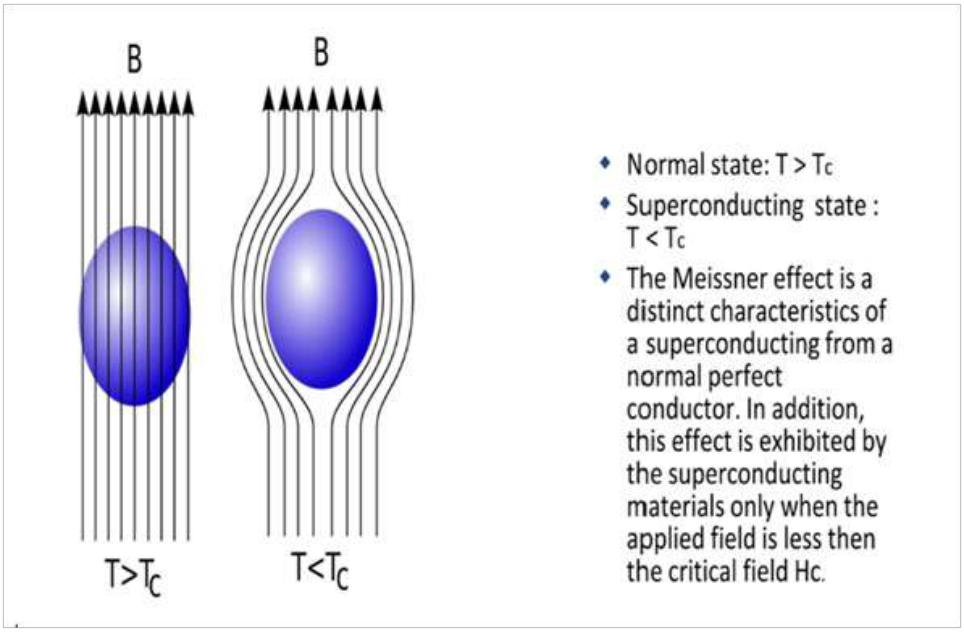


Figure 18: Meissner Effect.

In Type II superconductors, the Meissner effect can exist in a mixed state, where magnetic flux partially penetrates the material in quantized vortices while the rest of the superconductor remains flux-free.

One theoretical explanation for the Meissner effect is provided by the London equations. These equations demonstrate that the magnetic field inside a superconductor decays exponentially over a characteristic distance, known as the London penetration depth, typically ranging between 20 and 40 nm. The London penetration depth is a critical parameter for understanding how magnetic fields behave near the surface of superconductors.

Unit 2: Semiconductor Devices

Comprehensive Lecture Notes

Syllabus Coverage: Direct and indirect band gaps, p-n junction diode formation of depletion region, barrier potential, biasing, and I-V characteristics, optoelectronics devices-LEDs, laser diode. Basics of photovoltaics- photovoltaic effect, determination of efficiency of PV cell.

2.1 Direct and Indirect Band Gaps

The electrical and optical properties of semiconductors are largely determined by their energy band structures. The relationship between the energy (E) of an electron and its momentum (p) or propagation constant (k) is fundamental to understanding these properties.

According to quantum mechanics, the energy of a free electron is given by the parabolic equation:

$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \tag{11}$$

Where:

- m: Mass of an electron
- $\hbar$: Reduced Planck's constant ($h/2\pi$)

Because $E \propto k^2$, plotting Energy versus k yields a parabolic curve. The alignment of the lowest energy point of the conduction band and the highest energy point of the valence band on this $E-k$ diagram determines whether a semiconductor is "direct" or "indirect".

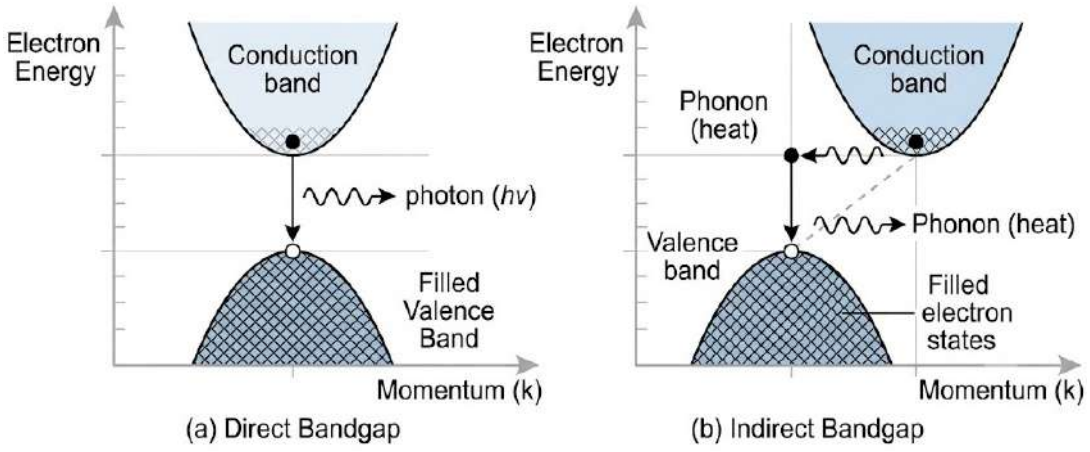


Figure 19: E-k Diagrams for Direct (a) and Indirect (b) Band Gap Semiconductors.

2.1.1 Direct Band Gap Semiconductors

In a direct band gap semiconductor, the conduction band minimum and the valence band maximum occur at the exact same value of crystal momentum (k).

- **Optical Transitions:** Because the momentum is the same in both states, an electron can transition directly between the conduction band and the valence band simply by absorbing or emitting a photon.
- **Efficiency:** This direct transition makes these materials highly efficient at emitting light, making them essential for manufacturing optoelectronic devices like LEDs and Laser Diodes.
- **Examples:** Gallium Arsenide (GaAs), Indium Arsenide (InAs), and Zinc Selenide (ZnSe).

2.1.2 Indirect Band Gap Semiconductors

In an indirect band gap semiconductor, the conduction band minimum and the valence band maximum occur at different values of crystal momentum (k vectors).

- **Optical Transitions:** An electron cannot shift from the lowest energy state in the conduction band to the highest energy state in the valence band solely by emitting a photon. The transition requires the involvement of a **phonon** (a quantum of crystal lattice vibration) to provide or absorb the necessary momentum.
- **Efficiency:** Because this is a two-step process (requiring both a photon and a phonon), the probability of radiative recombination is very low. Most energy is lost as heat instead of light.
- **Examples:** Silicon (Si), Germanium (Ge), and Lead Telluride (PbTe).

2.2 Theory of the P-N Junction Diode

The P-N junction is the fundamental building block of modern semiconductor electronics. A P-N junction diode allows electric current to pass essentially in only one direction, blocking it in the reverse direction.

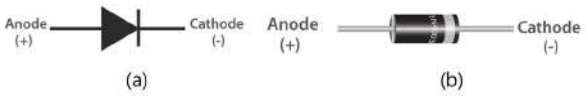


Figure 20: (a) Circuit Symbol (b) Photograph of commonly used diode

2.2.1 Construction and Carrier Concentration

A P-N junction is created when a single crystal of semiconductor is doped with acceptor impurities on one side (creating P-type material) and donor impurities on the other side (creating N-type material).

- **N-Type Region:** Contains a high concentration of free electrons (majority carriers) and positively charged donor ions.
- **P-Type Region:** Contains a high concentration of holes (majority carriers) and negatively charged acceptor ions.

2.2.2 Formation of the Depletion Region

The formation of the junction involves a dynamic balance between diffusion and drift.

1. **Diffusion:** Due to the severe concentration gradient, free electrons from the N-side diffuse across the junction to the P-side, while holes from the P-side diffuse to the N-side.
2. **Ion Uncovering:** As free electrons leave the N-side, they leave behind fixed, positively charged donor atoms. Conversely, as holes leave the P-side, they uncover fixed, negatively charged acceptor atoms.
3. **Space Charge Region:** This process builds a localized region of positive charge on the N-side and negative charge on the P-side. This region is depleted of mobile charge carriers and is therefore called the **depletion region**, space charge region, or transition region. Its physical width is roughly 500 Å.

2.2.3 Barrier Potential (V_B)

The exposed positive and negative ions create an internal electric field that points from the N-side to the P-side.

- **Equilibrium:** When sufficient donor and acceptor atoms are uncovered, the internal electric field is strong enough to completely repel further majority carrier diffusion.
- **Potential Difference:** This internal field creates a built-in voltage known as the barrier potential (V_B).
- **Typical Values:** The barrier potential is approximately **0.3 V for Germanium** and **0.7 V for Silicon**.
- **Temperature Dependence:** The barrier potential decreases by **2 mV** for every 1°C rise in temperature.

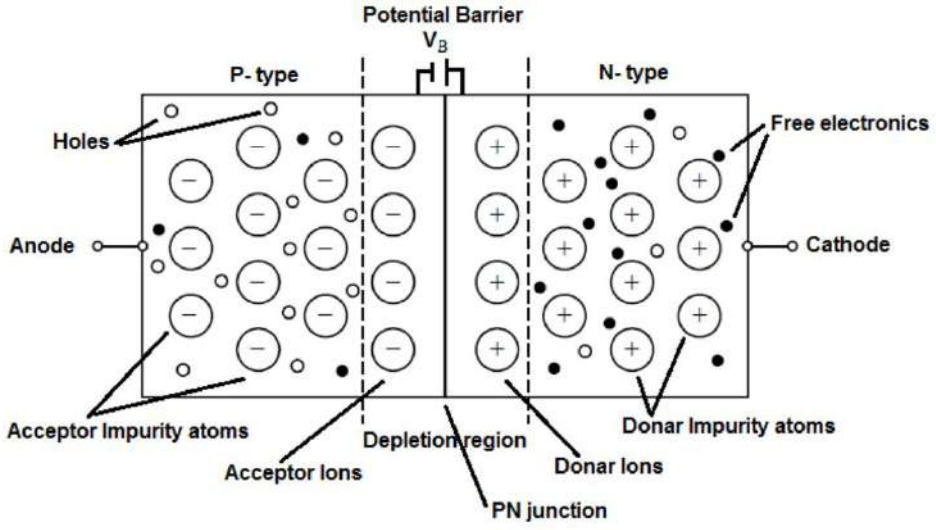


Figure 21: Unbiased P-N Junction showing the depletion region and barrier potential.

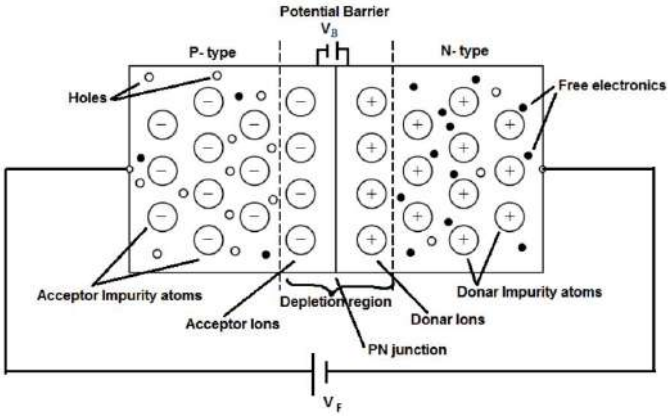
2.3 Biasing the P-N Junction

Biasing refers to applying an external DC voltage to the P-N junction to alter the width of the depletion region and control current flow.

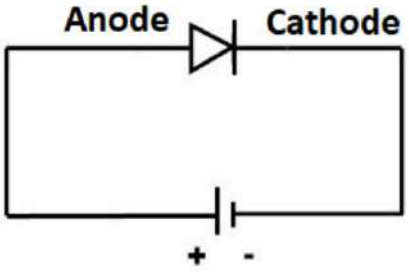
2.3.1 Forward Bias

In forward bias, the external voltage is applied to oppose the built-in barrier potential.

- **Connection:** The positive terminal of the battery connects to the P-region, and the negative terminal connects to the N-region.
- **Mechanism:** The positive terminal repels holes in the P-region toward the junction, while the negative terminal repels electrons in the N-region toward the junction.
- **Effect:** This forces carriers into the depletion region, effectively reducing the potential barrier and decreasing the depletion layer width.
- **Conduction:** Once the applied voltage exceeds the cut-in voltage (0.7 V for Si, 0.3 V for Ge), a large number of majority carriers easily cross the junction, resulting in a rapidly increasing forward current (I_F). The diode acts as a short circuit with very low resistance.



(a) Forward-biased P-N junction



(b) Circuit diagram of a forward-biased P-N junction diode

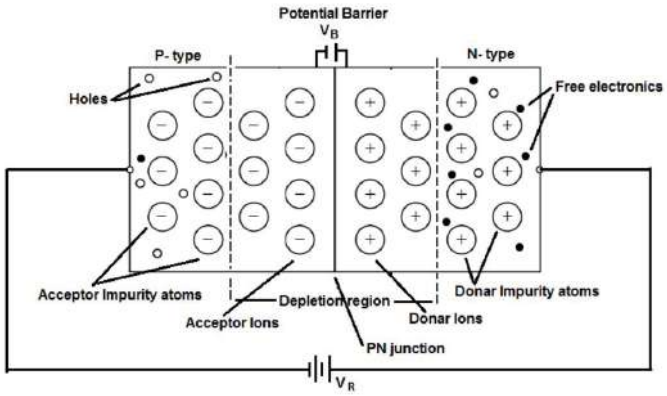
Figure 22: Forward-biased P–N junction and its equivalent circuit.

2.3.2 Reverse Bias

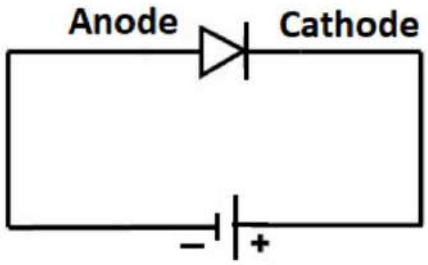
In reverse bias, the external voltage is applied in the same direction as the built-in potential, reinforcing it.

- **Connection:** The negative terminal connects to the P-region, and the positive terminal connects to the N-region.
- **Effect:** This widens the depletion region and increases the overall barrier potential, effectively preventing any majority carriers from crossing. The diode acts as an open circuit with high resistance.

- **Reverse Saturation Current (I_S):** Although majority carriers are blocked, the reinforced electric field assists *minority* carriers (thermally generated electrons in the P-side and holes in the N-side) across the junction. This produces a tiny, constant current (in μA) known as the reverse saturation current.



(a) Reversed-biased P-N junction



(b) Circuit diagram of a reverse-biased P-N junction diode

Figure 23: Reversed-biased P–N junction and its equivalent circuit.

2.4 I-V (Current-Voltage) Characteristics

The I-V characteristic curve is a graphical representation of the current flowing through a diode as a function of the applied voltage across its terminals.

2.4.1 Forward and Reverse Characteristics

- **Knee Voltage (Cut-in Voltage):** Under forward bias, when the voltage reaches approximately 0.7 V for Silicon (or 0.3 V for Germanium), the current increases exponentially. The voltage at which this sharp increase begins is called the knee voltage.
- **Leakage Region:** Under reverse bias, a small reverse saturation current (I_R) flows. It is largely independent of the applied voltage but highly dependent on temperature. For Silicon, I_R doubles for every 10°C rise in temperature.
- **Breakdown Region:** If the reverse voltage is increased beyond a critical threshold, the junction breaks down, resulting in a sudden and massive surge in current.

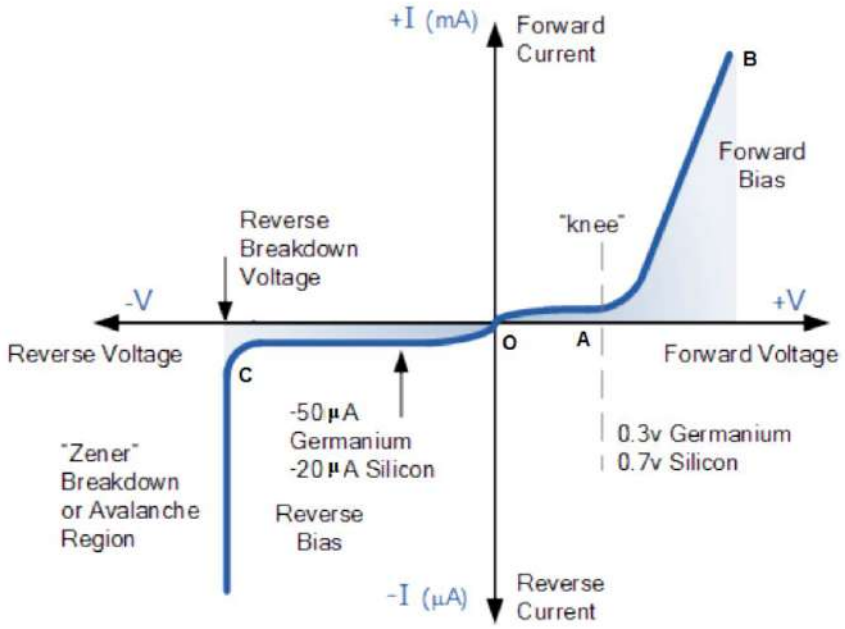


Figure 24: I-V Characteristic Curve of a typical P-N Junction Diode.

2.4.2 Junction Breakdown Mechanisms

1. **Zener Breakdown:** Occurs in heavily doped junctions with extremely narrow depletion regions (about 100 Å). A relatively low reverse voltage creates a massive electric field (e.g., 0.7×10^8 V/m) that directly pulls electrons out of their covalent bonds, creating a sharp increase in current.

2. **Avalanche Breakdown:** Occurs in lightly to moderately doped junctions with wider depletion regions. Minority carriers gain significant kinetic energy from the reverse electric field and collide with lattice atoms, knocking loose additional electrons, creating an "avalanche" multiplication of charge carriers.

2.4.3 The Diode Equation

The mathematical behavior of the I-V curve is described by the Shockley diode equation:

$$I = I_R \left(e^{\frac{qV}{nkT}} - 1 \right) \tag{12}$$

Where:

- I: Total current through the diode
- I_R: Reverse saturation current
- V: External applied voltage
- q: Electronic charge (1.602×10^{-19} C)
- k: Boltzmann's constant (1.38×10^{-23} J/K)
- T: Absolute temperature in Kelvin
- n: Ideality factor ($n = 1$ for Ge, $n = 2$ for Si)

2.5 Optoelectronic Devices: LEDs and Laser Diodes

2.5.1 Light Emitting Diode (LED)

An LED is a heavily doped P-N junction diode designed to convert electrical energy into light via a process called **electroluminescence**.

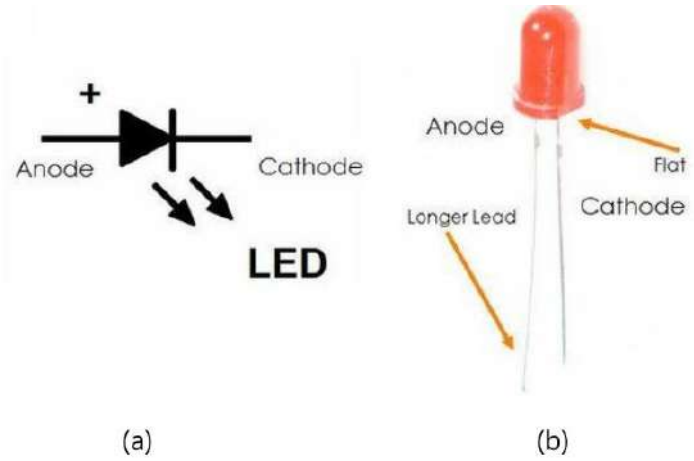
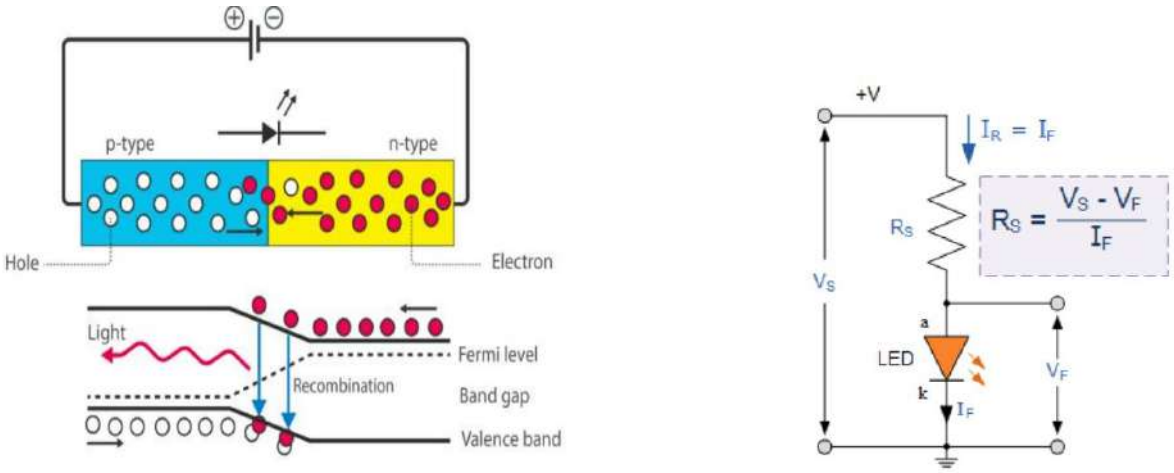


Figure 25: (a) Circuit symbol of a light-emitting diode (LED) and (b) photograph of an LED.

- **Working Principle:** LEDs operate strictly in forward bias. Electrons are injected into the N-region and holes into the P-region. When they cross the junction, free electrons in the conduction band recombine with holes in the valence band.
- **Light Emission:** In direct band gap materials (like GaAsP), this recombination releases energy predominantly as photons (light) rather than heat.
- **Circuit Protection:** LEDs operate at low voltages (1.6 V to 4.4 V) and low currents (10 to 50 mA). To prevent thermal destruction, a current-limiting series resistor (R_S) must be used in the circuit:

$$R_S = \frac{V_S - V_F}{I_F} \tag{13}$$



(a) Working principle of an LED

(b) LED circuit diagram

Figure 26: (a) Working principle of a light-emitting diode (LED), and (b) LED circuit diagram.

2.5.2 Laser Diode

While an LED emits light through spontaneous emission, a Laser Diode produces a coherent, monochromatic, and highly directional beam of light via **stimulated emission**.

- **Population Inversion:** For a laser diode to function, heavy forward biasing is used to pump a massive number of electrons into the conduction band, achieving population inversion.
- **Stimulated Emission:** When an electron naturally drops to the valence band, it releases a photon. If this photon strikes another excited electron, it stimulates that electron to drop down as well, releasing a second photon that is entirely identical to the first.
- **Optical Cavity:** The edges of the semiconductor crystal are cleaved and polished to act as mirrors. The photons reflect back and forth, amplifying the light exponentially before a laser beam escapes through a partially reflective facet.

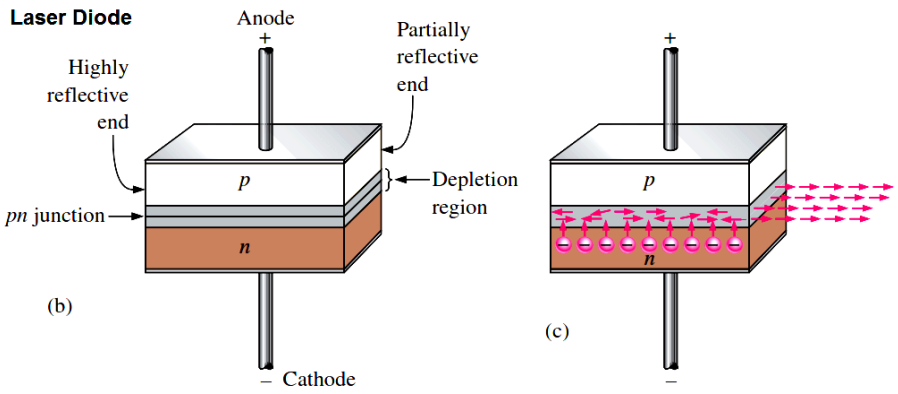


Figure 27: Structure and working principle of a laser diode.

2.6 Basics of Photovoltaics

A photovoltaic (PV) cell is an optoelectronic device that performs the reverse function of an LED: it converts light energy directly into electrical energy through the **photoelectric effect**.

2.6.1 Construction and Principle

- **Structure:** A standard PV cell consists of a thin N-type emitter layer on top, a thicker P-type base layer on the bottom, and a P-N junction sandwiched between them. The top surface features an anti-reflective coating to maximize light absorption.
- **Operating Mode:** PV cells operate inherently in a **reverse bias** condition without an external battery.
- **The Photovoltaic Effect:** Incident sunlight containing photons strikes the cell. If a photon's energy is greater than the semiconductor's bandgap energy ($h\nu > E_g$), it is absorbed, creating an electron-hole pair directly within the depletion region. The strong built-in electric field immediately separates these charge carriers, sweeping electrons toward the N-side and holes toward the P-side, generating a continuous Direct Current (DC) supply.

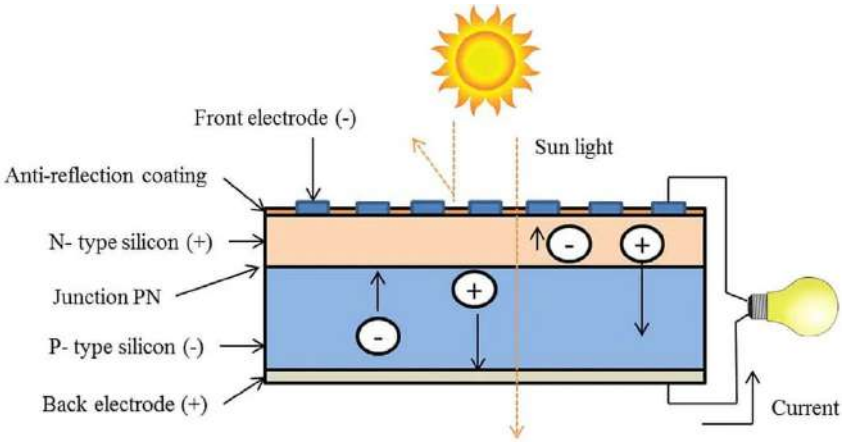


Figure 28: Cross-sectional Structure of a Photovoltaic Cell.

2.6.2 Determination of Efficiency of a PV Cell

The efficiency (η) of a solar cell measures how effectively the device converts incoming solar energy into usable electrical power.

To determine efficiency, three primary parameters are evaluated:

1. **Short Circuit Current (I_{sc}):** The maximum current that flows when the terminals are shorted (voltage across the cell is zero).

2. **Open Circuit Voltage (V_{oc}):** The maximum voltage across the terminals when the circuit is open (current is zero).

3. **Fill Factor (FF):** A metric evaluating the overall "squareness" of the I-V curve. It is the ratio of the actual maximum power output (P_{max}) to the theoretical maximum power ($I_{sc} \times V_{oc}$). A high-quality PV cell typically has a Fill Factor between **0.8 and 0.9**.

The overall efficiency equation is given as:

$$\eta = \frac{P_{max}}{P_{in}} = \frac{I_{sc} \times V_{oc} \times \text{FF}}{P_{in}} \tag{14}$$

Where P_{in} is the total incident solar power. Maximizing I_{sc}, V_{oc}, and Fill Factor is the primary goal of solar cell engineering.

Unit 3: Introduction to Quantum Physics

Comprehensive Lecture Notes

Wave-Particle Duality · Schrödinger Equation · Uncertainty Principle · Quantum Tunneling

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Syllabus Coverage: Limitations of classical mechanics (Qualitative) · de Broglie's hypothesis · Introduction to quantum computing (qualitative) · Wave particle duality · Physical significance of the wave function · Time-independent and dependent Schrodinger wave equations · Schrodinger equation for particle in one dimensional box · particle in finite potential · tunneling effect (Qualitative).

3.1 Inadequacies of Classical Physics (Qualitative)

Classical mechanics, governed by Newton's laws, and classical electromagnetism, formalized by Maxwell's equations, provide an excellent description of macroscopic systems. However, at the turn of the twentieth century, these theories failed dramatically when applied to atomic and subatomic systems.

The fundamental breakdown of classical mechanics can be attributed to its continuous view of energy and its strict separation between localized particles and propagating waves. Experimentally, this breakdown manifested in several key phenomena:

- **Blackbody Radiation:** Classical theory (Rayleigh-Jeans law) predicted that an ideal thermal radiator would emit infinite energy at short wavelengths, an unphysical result known as the *Ultraviolet Catastrophe*. This was resolved only when Max Planck postulated that energy exchange is discrete, or quantized.
- **The Photoelectric Effect:** Classical wave theory stated that the kinetic energy of emitted electrons should scale with light intensity and display a measurable time lag during low-intensity illumination. Experiments showed the energy depended strictly on light frequency with instantaneous emission.
- **Atomic Line Spectra:** Classical electrodynamics predicted that electrons orbiting a nucleus would continuously radiate energy and spiral rapidly into the core, making stable atoms impossible.

3.1.1 Blackbody Radiation and the Ultraviolet Catastrophe

One of the most glaring failures of classical physics occurred in predicting the spectral energy density of an ideal thermal radiator, or blackbody. According to classical thermodynamics and the Rayleigh-Jeans law, the energy density $u(\nu, T)$ at a frequency ν is proportional to the square of the frequency:

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} k_B T \tag{15}$$

As a result, as the frequency approaches the ultraviolet spectrum and beyond ($\nu \to \infty$), the predicted energy density approaches infinity. This unphysical divergence is known as the **Ultraviolet Catastrophe**.

This anomaly was resolved in 1900 by Max Planck, who assumed that the atomic oscillators in the cavity walls could only exchange energy in discrete packets, or *quanta*, given by $E = nh\nu$. This quantization led to the celebrated Planck's Radiation Law, which fits experimental data perfectly across all frequencies and naturally contains Wien's displacement traits and the localized peak shown below.

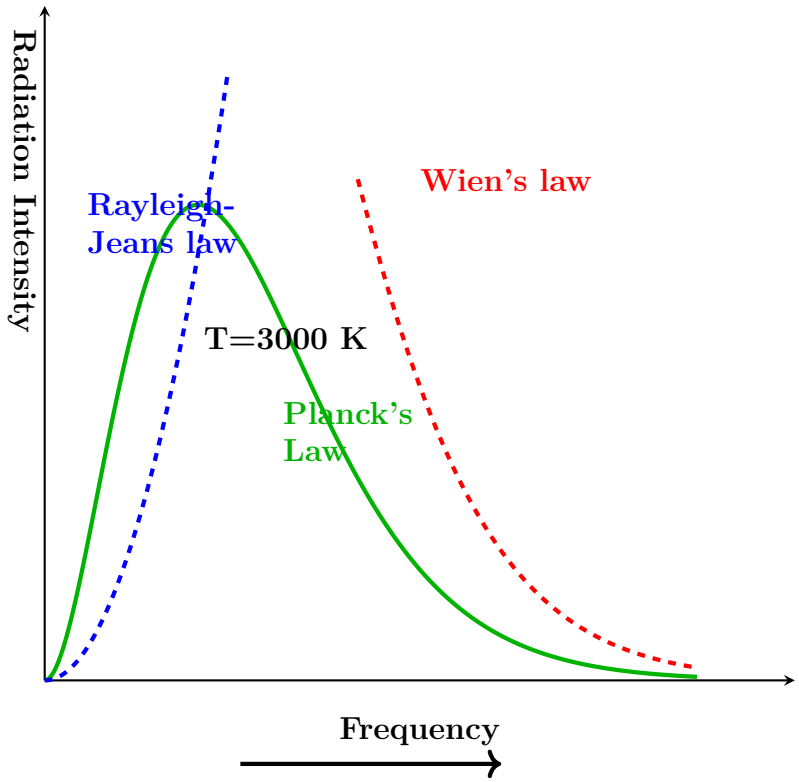


Figure 29: Comparison of spectral density distributions modeled exactly to the experimental frequency spectrum.

3.1.2 Wave Nature of Particles

Classical physics treats moving particles (such as electrons) purely as localized mass objects. This paradigm fails to predict quantum interference.

3.1.2.1 Young's Double Slit Experiment with Electrons

When a beam of mono-energetic electrons passes through two closely spaced sub-micron slits, they do not produce two simple linear bands on a detector screen. Instead, they form a clear, geometric interference pattern containing alternate intensity maxima and minima. This demonstrates that individual material particles propagate with wave characteristics.

According to Louis de Broglie's hypothesis, any moving particle has an associated matter wavelength given by:

$$\lambda = \frac{h}{p} \tag{16}$$

where:

- λ is the de Broglie wavelength,
- h is Planck's constant (6.626×10^{-34} J $\cdot$ s),
- $p = mv$ is the classical linear momentum of the particle.

This wave nature was conclusively validated by the **Davisson-Germer experiment**, where electrons scattered off a single crystal of nickel produced a diffraction pattern satisfying the Bragg condition, directly matching the de Broglie calculation.

3.1.3 Particle Nature of Light

Conversely, phenomena historically viewed as purely wave-like, such as light propagation, display localized particle-like traits when interacting with matter at microscopic scales.

3.1.3.1 Young's Double Slit Experiment with Light

When Young's double-slit experiment is performed using an ultra-attenuated light source, individual localized impacts register on the detector one by one. Each impact deposits a discrete packet of energy and momentum, behaving like a localized bullet rather than a smooth, continuous wave. The energy of each packet, or photon, is given by:

$$E = h\nu \tag{17}$$

where ν is the frequency of the radiation. Over time, as thousands of random photon strikes accumulate, a deterministic wave interference pattern emerges, illustrating the foundational probabilistic architecture of quantum theory.

3.1.3.2 Photoelectric Effect

Albert Einstein explained the photoelectric effect by treating light as a stream of photons. When a photon collides with an electron in a metal surface, it transfers its entire quantum of energy instantaneously. Part of this energy is spent overcoming the potential binding energy of the metal, known as the work function (W_0). The remaining energy becomes the maximum kinetic energy of the ejected photoelectron:

$$\text{K.E.}_{\cdot\max} = h\nu - W_0 = h\nu - h\nu_0 \tag{18}$$

where ν_0 represents the threshold frequency of the target material below which emission is physically impossible, regardless of beam intensity.

3.2 de Broglie's hypothesis

According to de-Broglie's concept, a moving particle always has a wave associated with it, and the motion of the particle is guided by that wave in similar manner as photon is controlled by a wave. Like electromagnetic waves, matter waves can also be propagated in a vacuum. Hence, they are not mechanical waves like sound waves. And unlike electromagnetic waves, they are also associated with the motion of electrically neutral bodies. Hence, they are not electromagnetic waves, but they are a new kind of wave.
According to de-Broglie, the wavelength of a wave associated with a moving particle depends upon the mass(m) of the particle and its velocity(v). The wavelength(λ) of a matter wave, or de Broglie wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \tag{19}$$

where h is the Planck's constant and p the momentum of the material particle. If E is the kinetic energy of the material particle,

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

From the above formula, the momentum of particle $p = \sqrt{2mE}$, therefore the de-Broglie wavelenth λ assocaited with a particle of mass m and kinetic energy, can E also be written as

$$\lambda = \frac{h}{\sqrt{2mE}} \tag{20}$$

If a charged particle carrying a charge q is accelerated through a potential difference of V volts, then the wavelength of the wave associated with this moving charged particle is obtained as follows:
The kinetic energy of the charged particle, $E = qV$, the de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mqV}} \tag{21}$$

for material particles, like neutrons, which possess Maxwellian distribution of velocities at absolute temperature T in thermal equilibrium, the average kinetic energy is given by

$$E = \frac{1}{2}mv^2{}_{rms} = \frac{3}{2}kT$$

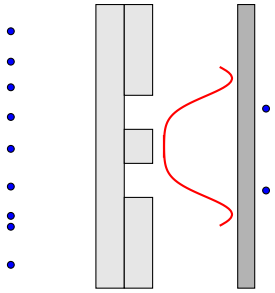
where k is the Boltzmann's constant. The wavelength of a wave associated with such a particle is given by

$$\lambda = \frac{h}{\sqrt{3mkT}} \tag{22}$$

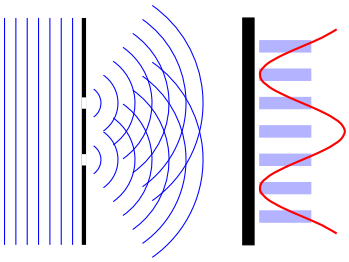
The equation 19 to 22 represents the de-Broglie's wavelength of a wave associated with various moving particles.

3.3 Wave-Particle Duality

The core synthesis of quantum mechanics is that energy and matter cannot be categorized exclusively as either waves or particles. They display both behaviors concurrently. The experimental configuration dictates which characteristic dominates: propagation through space yields wave-like phase interference, while localized absorption or collision yields particle-like discrete energy transfer.



(a) Classical Particle Profile



(b) Quantum Wave Interference

Figure 30: Double slit intensity profiles distinguishing localized classical particles from overlapping quantum matter waves.

3.4 Mathematical Formalism of Quantum Mechanics

3.4.1 Physical Significance of the Wave Function (ψ)

In quantum mechanics, the mechanical state of a system is completely specified by a complex-valued state variable called the wave function, denoted as $\psi(x, y, z, t)$. Because ψ contains imaginary components, it cannot be directly measured.

The physical meaning of the wave function was provided by Max Born's probabilistic interpretation: the square of the absolute magnitude of the wave function represents a spatial probability density. For a one-dimensional system, the quantity:

$$P(x,t) = |\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t) \tag{23}$$

defines the probability per unit length of finding the particle at coordinate x at time t, where ψ^* is the complex conjugate. The probability of locating the particle inside a finite domain between boundaries a and b is:

$$P(a \leq x \leq b) = \int_a^b |\psi(x,t)|^2 dx \tag{24}$$

For a wave function to represent a real physical system, it must be single-valued, continuous, have a continuous first derivative, and satisfy the **normalization condition**, which states the particle must exist somewhere in space:

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 dx = 1 \tag{25}$$

3.4.2 The Schrödinger Wave Equations and Significance

The foundational framework governing how wave functions evolve and distribute is given by the equations formulated by Erwin Schrödinger.

3.4.3 Time Independent Schrödinger Equation (TISE)

Let us assume the time independent wave function as:

$$\Psi = \Psi_0 e^{ikx} \tag{26}$$

where Ψ_0 is amplitude, k is wave number, i is the imaginary unit, and x is the position. Differentiating equation 26 with respect to x: $\dfrac{\partial \Psi}{\partial x} = (ik)\Psi_0 e^{ikx}$

$$\text{Or} \qquad \frac{\partial \Psi}{\partial x} = (ik)\Psi \qquad \text{[using 26]}$$

$$\text{Or} \qquad \frac{\partial \Psi}{\partial x} = i\left(\frac{2\pi}{\lambda}\right)\Psi \qquad \left[k = \frac{2\pi}{\lambda}\right]$$

$$\text{Or} \qquad \frac{\partial \Psi}{\partial x} = i\left(\frac{2\pi p}{h}\right)\Psi \qquad \left[\lambda = \frac{h}{p}\right]$$

Solve for Momentum Operator p

$$\text{Or} \qquad p\Psi = -i\frac{h}{2\pi}\frac{\partial \Psi}{\partial x}$$

$$p = -i\frac{h}{2\pi}\frac{\partial}{\partial x} \tag{27}$$

3.4.3.1 Write Total Energy in Terms of kinetic energy and potential energy

As we know, energy (E) can be written in terms of kinetic energy and potential energy as:

$$E\Psi = \frac{p^2}{2m}\Psi + V\Psi \tag{28}$$

Substitute the Momentum Operator into the Energy Equation Using equation 27 in 28, we get:

$$E\Psi = -\frac{h^2}{8\pi^2 m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

Final form Time Independent Schrödinger Equation

$$\text{Or} \qquad \frac{\partial^2 \Psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0 \tag{29}$$

This is the time-independent Schrödinger wave Equation.

Significance of the TISE

- **Energy Eigenvalue Problem:** Structurally, the TISE is an eigenvalue equation ($\hat{H}\psi = E\psi$) where $\hat{H}$ is the Hamiltonian operator. Solving it yields the allowed discrete energy states (E) of a quantum system.
- **Stationary States:** Although the phase factor ($e^{-iEt/\hbar}$) ticks over time, it disappears when evaluating the real spatial probability density ($|\Psi(x,t)|^2 = |\psi(x)|^2$). Consequently, states solving the TISE feature static probability distributions over time, meaning they are *stationary*.

3.4.4 Time Dependent Schrodinger wave equation (TDSE)

Let us assume the time dependent wave function as: $\Psi = \Psi_0 e^{i(kx-\omega t)}$ ...(i) where Ψ_0 is amplitude, k is wave number, x is position, $\omega\,(= 2\pi\nu)$ is angular frequency, and t is time.

Differentiating equation $\Psi = \Psi_0 e^{i(kx-\omega t)}$ with respect to x:

$$\frac{\partial \Psi}{\partial x} = -(ik)\Psi$$

$$\text{Or} \quad \frac{\partial \Psi}{\partial x} = i\left(\frac{2\pi}{\lambda}\right)\Psi \qquad \left[k = \frac{2\pi}{\lambda}\right]$$

$$\text{Or} \quad \frac{\partial \Psi}{\partial x} = i\left(\frac{2\pi p}{h}\right)\Psi \qquad \left[\lambda = \frac{h}{p}\right]$$

$$\text{Or} \quad p\Psi = -i\frac{h}{2\pi}\frac{\partial \Psi}{\partial x}$$

$$\text{Or} \quad p = -i\frac{h}{2\pi}\frac{\partial}{\partial x} \qquad \text{...(ii)}$$

Differentiating equation (i) with respect to t:

$$\frac{\partial \Psi}{\partial t} = -(i\omega)\Psi_0 e^{i(kx-\omega t)}$$

$$\frac{\partial \Psi}{\partial t} = -(i\omega)\Psi \qquad \text{[Using (i)]}$$

$$\text{Or} \quad \frac{\partial \Psi}{\partial t} = -i(2\pi\nu)\Psi \qquad [\omega = 2\pi\nu]$$

$$\text{Or} \quad h\frac{\partial \Psi}{\partial t} = -i(2\pi h\nu)\Psi \qquad \text{[Multiplying } h \text{ both sides]}$$

$$\text{Or} \quad E\Psi = i\frac{h}{2\pi}\frac{\partial \Psi}{\partial t} \qquad [E = h\nu]$$

$$\text{Or} \quad E = i\frac{h}{2\pi}\frac{\partial}{\partial t} \qquad \text{...(iii)}$$

Substituting the values of p and E from equations (ii) and (iii) into equation (15), we get the **time-dependent Schrödinger Wave Equation**:

$$i\frac{h}{2\pi}\frac{\partial \Psi}{\partial t} = -\frac{h^2}{8\pi^2 m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi \tag{30}$$

This is the time-dependent Schrödinger wave equation.

Significance of the TDSE

- **Quantum Equations of Motion:** The TDSE plays the exact same role in quantum mechanics that Newton's second law ($F = ma$) plays in classical mechanics. It represents the fundamental law governing how a state changes over time.
- **Conservation of Probability:** The inclusion of the imaginary unit i guarantees that the time evolution operator is strictly *unitary*, enforcing that total probability remains exactly equal to 1 indefinitely.

3.5 Applications of the Schrödinger Equation

3.5.1 Rigorous Derivation: Particle in an Infinite 1D Potential Well

A foundational application of the Time-Independent Schrödinger Equation is a subatomic particle confined within a one-dimensional box with infinitely impenetrable boundaries. Consider a particle of mass m restricted to slide freely along the x-axis between positions $x = 0$ and $x = a$. The static spatial potential energy function $V(x)$ is defined mathematically by:

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < a \quad \text{(Region II)} \\ \infty & \text{for } x \leq 0 \text{ or } x \geq a \quad \text{(Regions I and III)} \end{cases} \tag{31}$$

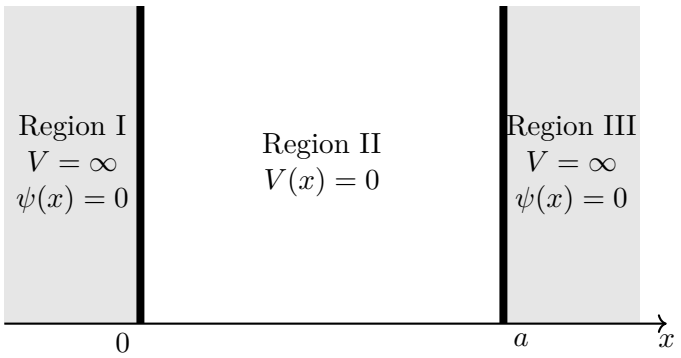


Figure 31: Boundary map of a particle inside a one-dimensional infinite potential well configuration.

3.5.1.1 Step 1: Setting up the Ordinary Differential Equation

In Regions I and III ($x \leq 0$ and $x \geq a$), the potential energy is infinite ($V = \infty$). For the physical wave function to prevent the total energy expectation value from blowing up, it must be zero everywhere in those regions:

$$\psi_{\text{I}}(x) = 0 \quad \text{and} \quad \psi_{\text{III}}(x) = 0 \tag{32}$$

Inside the box (Region II, where $0 < x < a$), the potential energy drops to zero ($V(x) = 0$). Substituting this criteria into the Time-Independent Schrödinger Equation yields:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x) \implies \frac{d^2\psi(x)}{dx^2} + \left(\frac{2mE}{\hbar^2}\right)\psi(x) = 0 \tag{33}$$

We define a constant real wave number parameter k such that $k^2 = \frac{2mE}{\hbar^2}$. The standard form of the second-order linear homogeneous differential equation is:

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0 \tag{34}$$

The general solution for this differential equation is a linear combination of harmonic sinusoids:

$$\psi(x) = A\sin(kx) + B\cos(kx) \tag{35}$$

where A and B are arbitrary constants determined by physical boundary matching constraints.

3.5.1.2 Step 2: Applying Boundary Conditions

Because the wave function represents a continuous physical probability distribution, it must match smoothly at the boundaries of the well. This means $\psi(x)$ cannot change discontinuously at $x = 0$ or $x = a$:

1. **First Boundary Condition (at $x = 0$):**

$$\psi(0) = 0 \implies A\sin(k \cdot 0) + B\cos(k \cdot 0) = 0 \tag{36}$$

Directly simplifying yields:

$$B = 0 \tag{37}$$

Substituting $B = 0$ back into Equation (35) leaves:

$$\psi(x) = A\sin(kx) \tag{38}$$

2. **Second Boundary Condition (at $x = a$):**

$$\psi(a) = 0 \implies A\sin(ka) = 0 \tag{39}$$

For a non-trivial solution ($A \neq 0$), the trigonometric term itself must collapse to zero:

$$\sin(ka) = 0 \implies ka = n\pi \quad \text{for } n \in \{1, 2, 3, \dots\} \tag{40}$$

Solving for the allowed values of the wave number gives:

$$k_n = \frac{n\pi}{a} \tag{41}$$

3.5.1.3 Step 3: Deriving Quantized Energy Eigenvalues

By equating our boundary expression for k_n to the definition of k^2, we isolate the energy values E_n:

$$k_n^2 = \frac{2mE_n}{\hbar^2} = \left(\frac{n\pi}{a}\right)^2 \implies E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} \tag{42}$$

Substituting $\hbar = \frac{h}{2\pi}$ provides the energy values in terms of Planck's standard constant:

$$E_n = \frac{n^2\pi^2}{2ma^2}\left(\frac{h}{2\pi}\right)^2 = \frac{n^2h^2}{8ma^2} \tag{43}$$

This indicates that the energy of the confined particle cannot take on a continuous spectrum; it is quantized into discrete levels proportional to n^2.

3.5.1.4 Step 4: Wave Function Normalization

To evaluate the absolute amplitude constant A, we enforce the Born normalization condition:

$$\int_0^a |\psi_n(x)|^2 dx = 1 \implies \int_0^a A^2 \sin^2\left(\frac{n\pi x}{a}\right) dx = 1 \tag{44}$$

Using the trigonometric identity $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$, we expand the integral expression:

$$A^2 \int_0^a \frac{1}{2}\left[1 - \cos\left(\frac{2n\pi x}{a}\right)\right] dx = 1 \tag{45}$$

$$\frac{A^2}{2}\left[x|_0^a - \frac{a}{2n\pi}\sin\left(\frac{2n\pi x}{a}\right)\Big|_0^a\right] = 1 \tag{46}$$

Evaluating at limits 0 and a:

$$\frac{A^2}{2}\left[(a-0) - \frac{a}{2n\pi}(\sin(2n\pi) - \sin(0))\right] = 1 \tag{47}$$

Since $\sin(2n\pi) = 0$ for all integers n, the entire oscillating boundary term drops out:

$$\frac{A^2}{2}[a] = 1 \implies A^2 = \frac{2}{a} \implies A = \sqrt{\frac{2}{a}} \tag{48}$$

Thus, the complete set of normalized spatial eigenfunctions solving the infinite potential well problem is:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \tag{49}$$

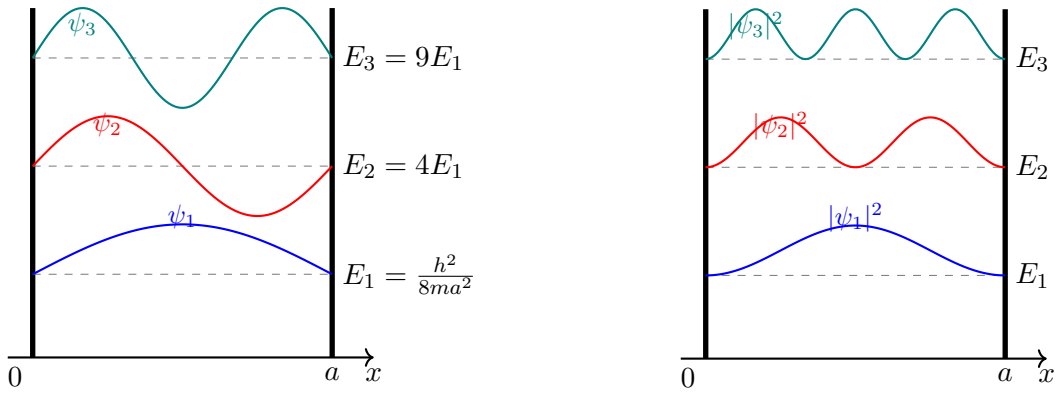


Figure 32: Side-by-side graphical mapping of the normalized wavefunctions $\psi_n(x)$ (left) and corresponding spatial localized probability distribution densities $|\psi_n(x)|^2$ (right) scaled across quadratic energy offsets inside an infinite potential well.

The Finite Potential Well

The finite potential well (or finite square well) is a fundamental model in quantum mechanics that describes a particle confined to a specific region of space by potential barriers of a finite height, V_0.

Unlike the idealized infinite potential well where the particle is strictly trapped, the finite well introduces the quantum phenomenon of **wavefunction penetration**. Because the barrier is not infinitely high, the particle has a non-zero probability of being found *inside* the classically forbidden barrier regions, decaying exponentially.

Potential Function and Regions

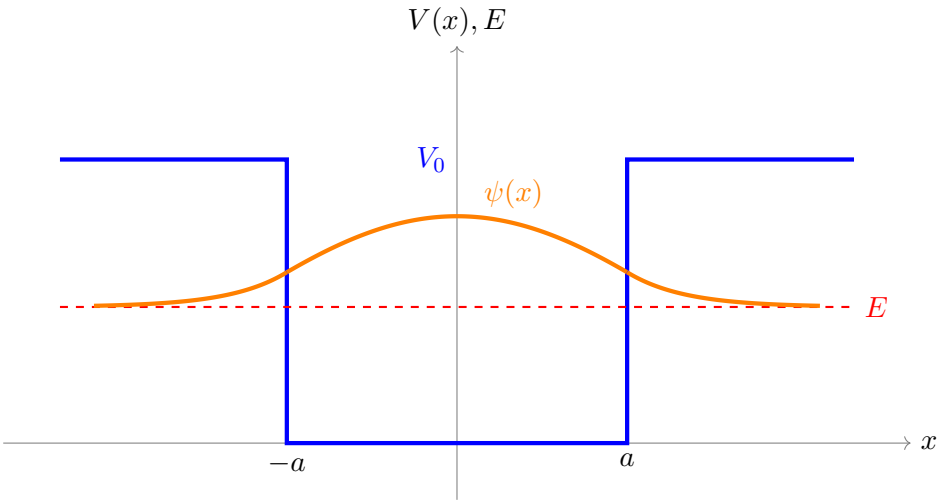
The potential energy function $V(x)$ is typically defined as:

$$V(x) = \begin{cases} V_0 & \text{for } x < -a \quad \text{(Region I)} \\ 0 & \text{for } -a \leq x \leq a \quad \text{(Region II)} \\ V_0 & \text{for } x > a \quad \text{(Region III)} \end{cases} \tag{50}$$

For a particle with energy $E < V_0$ (a bound state), the solutions to the Schrödinger equation are oscillatory (sine/cosine waves) inside the well and exponentially decaying outside the well.

Illustration

The figure below illustrates the potential well $V(x)$, a discrete energy level E, and the corresponding ground state wavefunction $\psi(x)$ demonstrating penetration into the barrier.



3.6 Quantum Tunneling Effect (Qualitative Discussion)

Quantum tunneling is a purely quantum-mechanical phenomenon wherein a microscopic particle penetrates and propagates through a finite potential energy barrier (V_0) despite possessing a total energy strictly less than the barrier height ($E < V_0$). Classically, a particle in this state faces a hard turning point at the barrier wall; lacking the kinetic energy to clear the potential peak ($K.E. = E - V_0 < 0$), it is reflected back with 100% certainty.

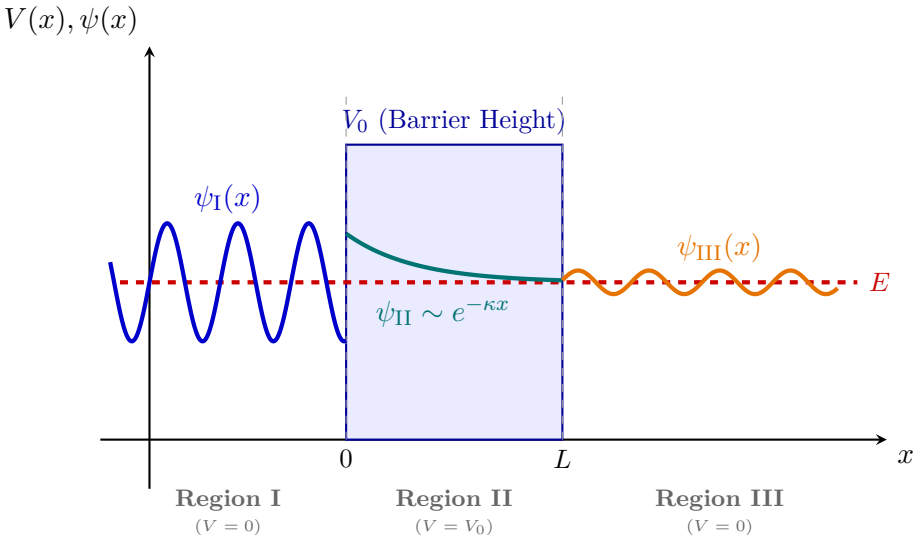


Figure 33: Mathematical phase profile of quantum tunneling through a 1D potential barrier of width L, demonstrating the exponential match between oscillating states.

In the quantum domain, matter behaves as a spatially distributed wave packet. When this propagating wave encounters a step up in potential energy that exceeds its total mechanical energy, the wave function cannot instantly collapse to zero due to physical continuity requirements. Instead, it transitions into an **evanescent wave** inside the barrier, decaying exponentially across the forbidden region.

If the spatial width of the barrier (L) is thin enough, the exponential tail of the wave function reaches the opposite interface before dropping entirely to zero. It then matches continuously with a free, oscillating wave solution on the other side. As a result, the particle has a non-zero probability—known as the **Transmission Coefficient (T)**—of emerging on the far side of the

barrier with its original energy intact, though with a drastically reduced probability amplitude.

The absolute likelihood of a particle tunneling through a classic barrier depends exponentially on two critical physical properties:

- **Barrier Thickness (L):** Because the internal wave function decays exponentially, even a small linear increase in the barrier width causes an order-of-magnitude drop in transmission probability.
- **Particle Mass (m) and Barrier Height ($V_0 - E$):** The exponential decay constant depends directly on the square root of the particle's mass ($\sqrt{m}$) and the potential deficit ($\sqrt{V_0 - E}$). This means light particles (like electrons) tunnel very efficiently, whereas heavier particles (such as protons or macroscopic items) experience such rapid exponential attenuation that their tunneling probability is practically zero.

3.6.0.1 Real-World Examples

1. Scanning Tunneling Microscopy (STM): The extreme sensitivity of the quantum tunneling probability to tiny distance variations forms the functional foundation of the Scanning Tunneling Microscope. An atom-sized metallic tip is positioned extremely close to a conducting sample surface. When a small bias voltage is applied, electrons tunnel across the empty, classically forbidden vacuum gap separating the tip and the sample, generating a measurable "tunneling current."

Because the vacuum barrier width changes by only a single atomic diameter, the resulting tunneling current scales up or down by a factor of nearly a thousand. By using feedback loops to adjust the tip height and keep the current constant, the microscope tracks and maps individual surface atoms with incredible sub-nanometer spatial resolution.

2. Alpha Decay in Nuclear Physics: Quantum tunnelling provides the definitive explanation for how alpha particles (He^{2+} nuclei) escape heavy radioactive elements, such as Uranium-238. Structurally, an alpha particle is bound inside the atomic nucleus by an incredibly deep potential well created by the strong nuclear force. Surrounding this well is a massive potential energy barrier driven by electrostatic repulsion.

Classically, the alpha particle has only about 4 to 9 MeV of kinetic energy, which is far lower than the roughly 25 MeV needed to scale the electrostatic barrier. However, by acting as a quantum wave packet that bounces repeatedly against the nuclear boundary walls, the alpha particle eventually tunnels through the forbidden potential wall, allowing the nucleus to undergo radioactive decay.

3.7 Introduction to Quantum Computing

Quantum computing leverages the principles of quantum mechanics to process information in ways that are impossible for classical computers.

Instead of classical bits that store binary states (0 or 1), quantum computers use quantum bits, or **qubits**. A qubit can exist in a linear combination of both states simultaneously through **superposition**: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Furthermore, quantum systems use **entanglement**, an intrinsically quantum correlation where the state of one qubit depends instantly on the state of another, regardless of distance. By utilizing superposition and entanglement alongside **quantum interference**, quantum algorithms can evaluate vast problem spaces concurrently. This provides exponential speedups for specialized tasks like integer factorization (Shor's Algorithm) and database searching (Grover's Algorithm).

3.7.1 Fundamental Differences between classical and quantum computing

Feature	Classical Computing	Quantum Computing
1. Basic Unit of Information	Uses **bits** (binary digits).	Uses **qubits** (quantum bits).
2. State	A bit can exist in only one state at a time: strictly **0** or **1**.	A qubit can exist as 0, 1, or both simultaneously (a state known as **superposition**).
3. Processing Approach	Processes data sequentially or in parallel through deterministic logic gates (AND, OR, NOT).	Explores multiple computational paths simultaneously using quantum interference.
4. Scaling Power	Scaling is **linear**. Adding one more bit adds one more unit of processing power (n).	Scaling is **exponential**. Adding one more qubit doubles the computational state space (2^n).
5. Core Mechanism	Relies on classical physics and electrical voltages (transistors switching on and off).	Relies on quantum mechanics principles, specifically superposition, entanglement, and interference.
6. Link Between Units	Bits operate independently of one another.	Qubits can be **entangled**, meaning the state of one qubit instantly correlates with the state of another, regardless of distance.
7. Error Rates & Stability	Highly stable. Error rates are extremely low and easily corrected with simple redundancy.	Highly unstable (**decoherence**). Qubits are incredibly sensitive to temperature and electromagnetic noise, requiring complex error correction.
8. Operating Environment	Can operate at normal room temperatures.	Usually requires extreme environments, such as temperatures near absolute zero (-273°C), to keep qubits stable.
9. Best Use Cases	Excellent for everyday tasks, databases, web browsing, word processing, and standard math.	Ideal for complex optimization problems, simulating molecules for drug discovery, cryptography, and complex system modeling.
10. Measurement Outcome	Reading a bit's state is definitive and does not change the data.	Measuring a qubit forces its superposition to collapse into a definite state (0 or 1), permanently changing it.

3.7.2 Visualizing the Basic Units: Bit vs. Qubit

Figure 34 contrasts a classical bit, which is restricted to two discrete states, with a quantum qubit (represented by a Bloch sphere), which can exist anywhere in a continuous state space.

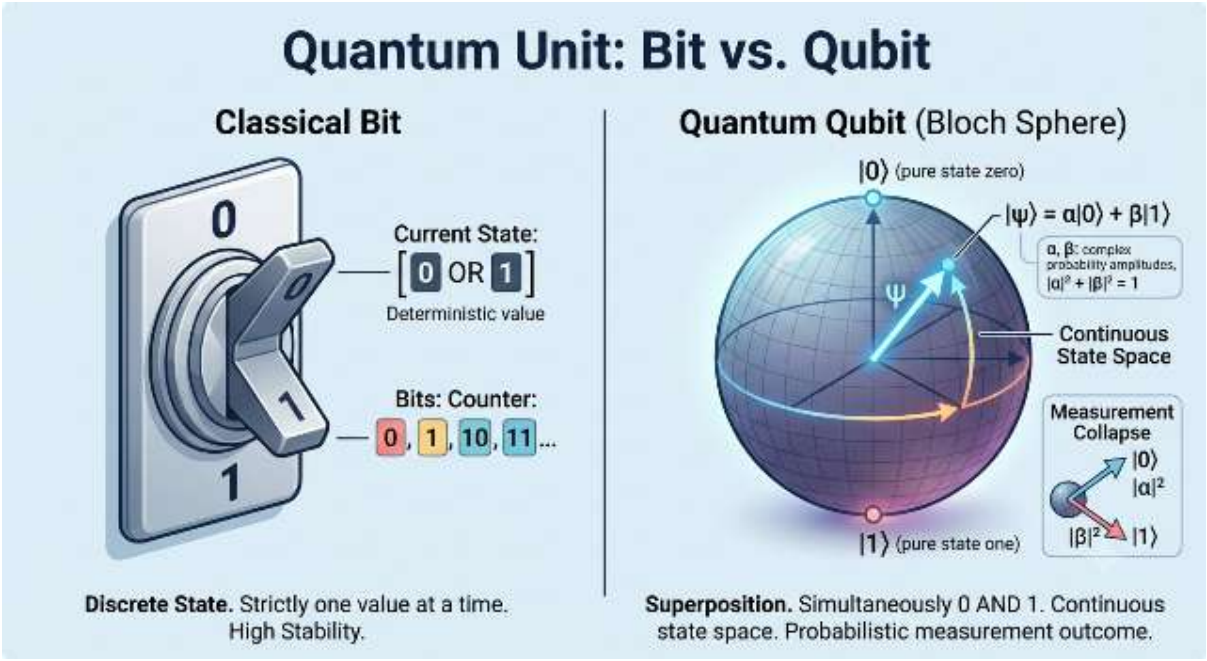
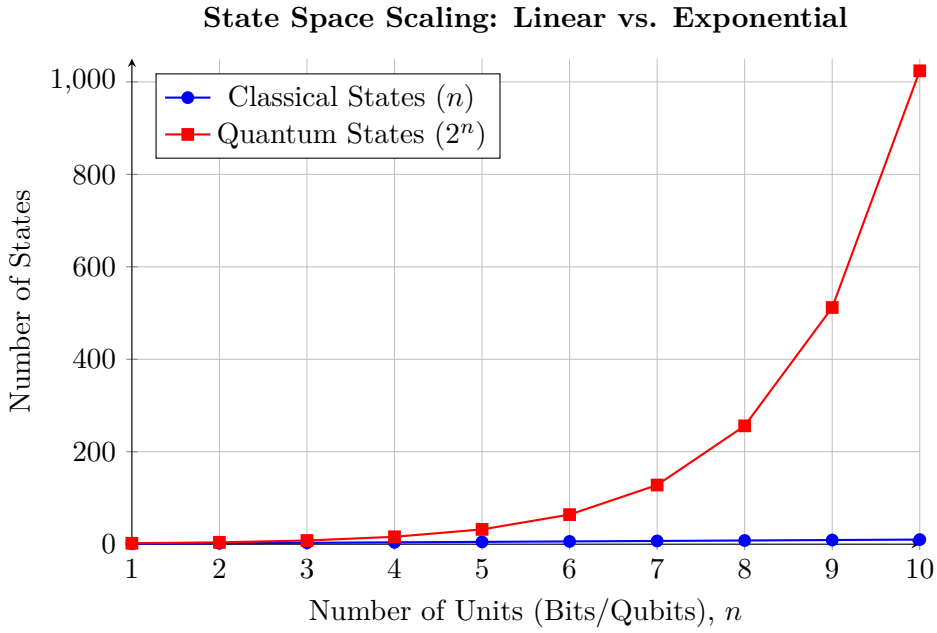


Figure 34: Comparison of a discrete classical bit switch and the continuous state space of a quantum qubit on a Bloch Sphere.

3.7.3 Visualizing Computational Power Scaling

The graph below demonstrates the exponential scaling of quantum state space (2^n) compared to the linear scaling of classical state space (n) as units of information are added.



Unit 4: Photonics and Optical Communication

Comprehensive Lecture Notes

Syllabus Coverage: Spontaneous and stimulated emission of radiations, Einstein's coefficients, metastable state, population inversion, lasing action, solid state laser (Ruby laser), gas laser (He-Ne laser), semiconductor laser and applications. Principle and construction of an optical fiber, acceptance angle, numerical aperture, types of optical fibers, and applications.

The lowest energy level for an individual atom occurs when its electrons are all in the nearest possible orbits to its nucleus; this energy level is called the "Ground state". When one or more of an atom's electrons have absorbed energy, they can move to outer orbits, and the atom is then referred to as being excited, and that energy level is called as "Excited state." Excited states are generally not stable; as electrons drop from higher-energy to lower-energy levels, they emit the extra energy as light.

Normally atoms are found in the ground state, it is the natural tendency of every system to achieve the state of least energy. At thermal equilibrium, atoms continuously jump from the ground to an excited state (provided the thermal energy is enough to excite the atoms), deexcited back, and so on. At this thermal equilibrium, the total number of atoms transiting from ground to excited energy is equal to the de-excited atoms.

4.1 Interaction of Radiation with matter

1. **Induced Absorption or Stimulated Absorption:** An atom is in the ground state with energy E_1 absorbs a photon of energy $h\nu$ and goes to the excited state with energy E_2 as shown in Fig. This transition is known as stimulated absorption or induced absorption or simply absorption. Here the energy difference is given as $(E_2–E_1) = h\nu$.

If there are many number of atoms in the ground state then each atom will absorb the energy from the incident photon and goes to the excited state then,

The rate of absorption (R_{12}) is proportional to,

$$\begin{aligned} R_{12} &\propto \text{Energy density of incident radiation } (u_\nu) \\ &\propto \text{No. of atoms in the ground state } (N_1) \\ R_{12} &= B_{12} u_\nu N_1 \end{aligned}$$

Where, B_{12} is a constant which gives the probability of absorption transition perunit time.

2. **Spontaneous emission:** The natural tendency of an atom is to seek out the lowest energy configuration.The excited atoms do not stay in the excited state for longer time but tend to return tothe lower state by giving up the excesses energy $h\nu$ as shown in fig. The atom in theexcited state E_2 returns to the ground state E_1 by emitting a photon of energy $h\nu$without any external energy. Such emission of radiation not initiated by any externalinfluence is called spontaneous emission. This emission is uncontrollable.

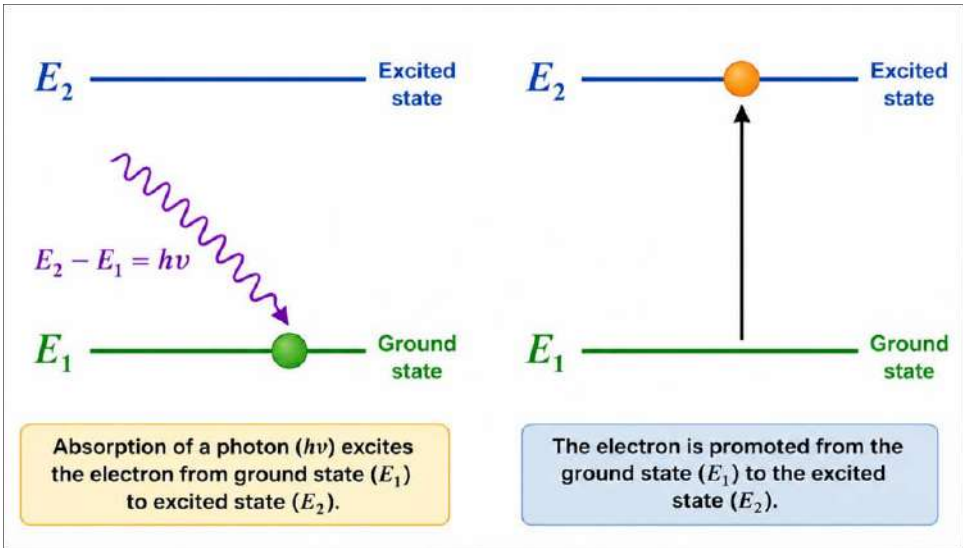


Figure 1: Schematic of stimulated absorption

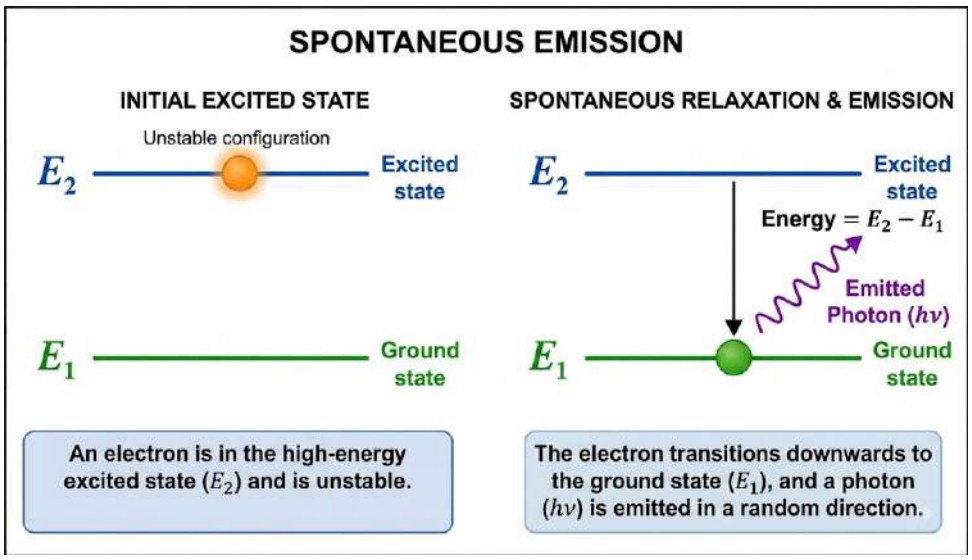


Figure 2: Schematic of spontaneous emission

The rate of spontaneous emission $R_{21}(Sp)$ is proportional to number of atoms ormolecules present in the excited state.

$$\text{i.e } R_{21}(Sp) \propto N_2$$
$$R_{21}(Sp) = A_{21}N_2$$

Where, A_{21} is a constant which gives the probability of spontaneous emissiontransitions per unit time.

3. **Stimulated emission**

The atom in the excited state E_2 as shown in fig. A photon of energy $h\nu$ canstimulate the atom to move to its ground state. During this process the atom emitsan additional photon whose energy is also $h\nu$. As the emission is stimulated byexternal photon, this process is known as stimulated emission.

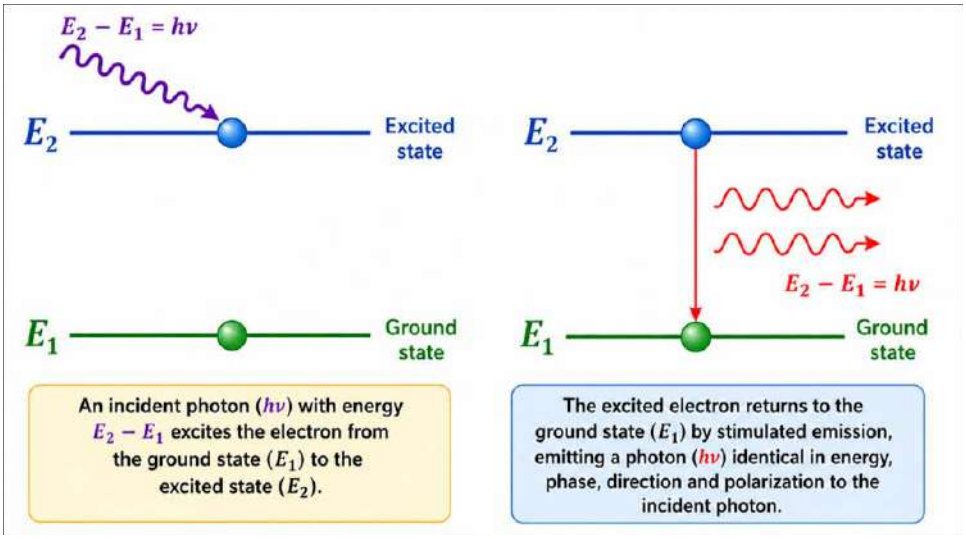


Figure 3: Schematic of stimulated emission

The rate of stimulated emission $R_{21}(St)$ is proportional to,

$$\begin{aligned} R_{21} &\propto \text{Energy density of incident radiation } (u_\nu) \\ &\propto \text{No. of atoms in the excited state } (N_2) \\ \text{i.e } R_{21}(St) &\propto u_\nu N_2 \\ R_{21}(St) &= B_{21} u_\nu N_2 \end{aligned}$$

where B_{21} is a constant which gives the probability of stimulated emissiontransitions per unit time.

4.2 Einstein's A and B Coefficient:

(Relation between Einstein's coefficient and Energy density of radiation)

Einstein's theory of absorption and emission of light by an atom is based onPlanck's theory of radiation.Consider two energy levels E_1 and E_2 having number of atoms N_1 and N_2respectively, as shown in figure.

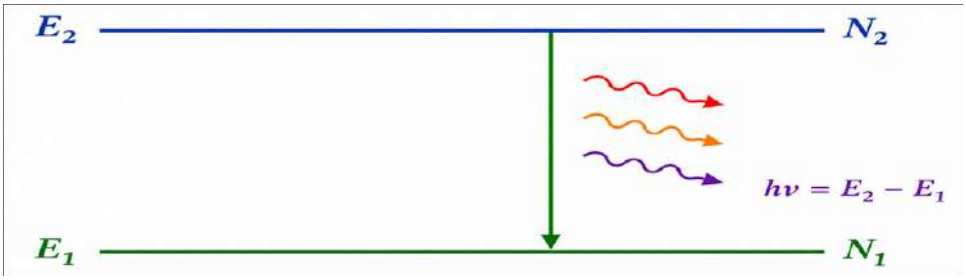


Figure 4: Lasing action

According to Einstein's theory,

- Rate of Stimulated absorption is, $R_{12} = B_{12} u_\nu N_1$
- Rate of Spontaneous Emission is, $R_{21}(Sp) = A_{21} N_2$
- Rate of Stimulated Emission is, $R_{21}(St) = B_{21} u_\nu N_2$

Where, A and B represents the Spontaneous and Stimulated processrespectively. u_ν is the energy density of radiation.

At thermal equilibrium, The rate of absorption = The rate of emission

$$B_{12}u_\nu N_1 = A_{21}N_2 + B_{21}u_\nu N_2 \tag{1}$$

$$\begin{aligned} u_\nu[B_{12}N_1 - B_{21}N_2] &= A_{21}N_2 \\ u_\nu &= \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2} \\ u_\nu &= \frac{A_{21}}{B_{12}\left(\frac{N_1}{N_2}\right) - B_{21}} \end{aligned} \tag{2}$$

Under thermal equilibrium, the population of energy levels obeys theBoltzmann's distribution law. We know from Boltzmann distribution law,

$$\begin{aligned} N_1 &= N_0 e^{-E_1/kT} \\ N_2 &= N_0 e^{-E_2/kT} \end{aligned}$$

Where, K is the Boltzmann constant, T is the absolute temperature andN_0 is the number of atoms at absolute zero at equilibrium, we can write the ratio of population as follows,

$$N_1/N_2 = e^{(E_2-E_1)/kT} \tag{1}$$

Since, E_2–$E_1 = h\nu$, we have

$$N_1/N_2 = e^{h\nu/kT} \tag{3}$$

Substitute eq(3) in eq(2), we get

$$u_\nu = \frac{A_{21}}{B_{12}e^{h\nu/kT} - B_{21}} \quad \text{or} \quad u_\nu = \frac{A_{21}/B_{21}}{(B_{12}/B_{21})e^{h\nu/kT} - 1} \tag{4}$$

Comparing eq. (4) with the Planck's radiation law $u_\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$, we get

$$\begin{aligned} \frac{A_{21}}{B_{21}} &= \frac{8\pi h\nu^3}{c^3} \\ \frac{B_{12}}{B_{21}} &= 1 \end{aligned}$$

Above expressions, give the required relation between Einstein's coefficients.

Now, from eq. (4), we can write

$$\frac{A_{21}}{B_{21}u_\nu} = e^{h\nu/kT} - 1 \tag{4}$$

Let light of wavelength 600 nm at 1000 K, $h\nu/kT = 24$ Thus, $\frac{A_{21}}{B_{21}u_\nu} = e^{24} - 1 \approx 10^{10}$

This implies that for the visible range (even at high temperature, spontaneous emission is dominating over stimulated emission for the two-level energy system. Hence, for the ordinary light source, lasing action can't be achieved for the two-level laser system.

The coefficient of proportionality i.e. A_{12}, B_{12}, and B_{21} are called as Einstein coefficient of spontaneous emission, stimulated absorption and stimulated emission, respectively.

4.3 LASER

The word LASER is an acronym for "Light Amplification by Stimulated Emission of Radiation". It is a powerful monochromatic light source of collimated beam in which the light waves are highly coherent. The laser light has many superior features compared to conventional light source. Einstein introduced this concept in 1917. Dr. T.H. Maiman demonstrated the first laser namely the ruby laser in the year 1960.

When stimulated emission occurs below infrared region of electromagnetic spectrum, the term is known as MASER (Microwave Amplification by Stimulated Emission of Radiation). When stimulated emission occurs in infrared region and above infrared region ie in visible and UV region, the term is known as LASER.

The phenomenon of stimulated emission was first used by Townes in 1954, in development of MASER. In 1958, Shawlow and Townes showed that MASER principle can be extended into visible region, and in 1960, Maiman built the first laser which is known as Ruby laser.

4.3.1 Characteristics of LASER:

Laser differs from the ordinary light with respect to some properties. They are

- Monochromaticity
- Directionality
- Coherence
- Intensity

1. **Monochromaticity:** Laser beam is highly monochromatic. It emits single wavelength (one colour) because atoms or molecules are transition between two energy states. Hence, it possesses good spectra since range of laser beam wavelength ($\Delta\lambda$) is very narrow. But, ordinary light emits combination of wide range of wavelength (colors) because atoms or molecules are transition from several number of excited states to ground state, so it emits different energies, therefore it is polychromatic.

2. **Directionality or Divergence:** The light ray coming ordinary light source travels in all directions, but laser light travels in single direction. For example, the light emitted from torch spreads 1km distance, But the laser light spreads a few centimeters distance even it travels longer distance. The ordinary source emits light in all directions and its angular spread is 1 metre/metre. But the laser is highly directional and non- divergent and its angular spread is 1mm/metre.

The angular spread (Φ) or divergence is given by,

$$\Phi = (r_2 – r_1)/(D_2 – D_1) \text{ degree} \tag{2}$$

Where, D_1, D_2 are any two distances from the laser source emitted and r_1, r_2 are the radii of the beam spots at a distance D_1 and D_2, respectively.

3. **Coherence:** A predictable correlation of the amplitude and phase at any one point with other point is called coherence. The light from a source consists of wave pattern. These wave patterns when identical in phase and direction are called coherent. Laser has a high degree of coherence than the ordinary sources. The coherence of laser emission results in an extremely high power of 5×10^6 watt/m^2. A laser beam can be focused to a very small area of about 0.7 m diameter.

4. **Intensity:** Laser light is highly intense than the ordinary light. This is because of coherence and directionality of laser. The ordinary light spreads in all directions, so the intensity reaching the target is very less. But in the case of laser, due to high directionality the intensity

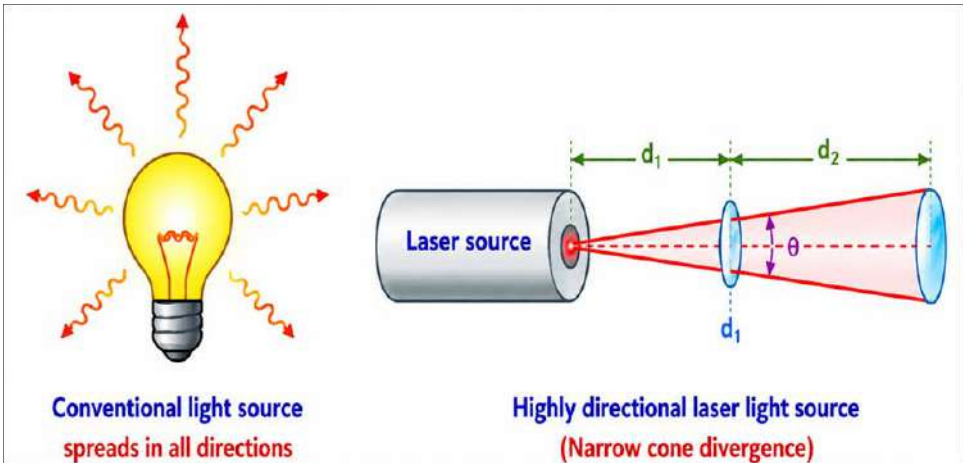


Figure 5: Directionality of laser light

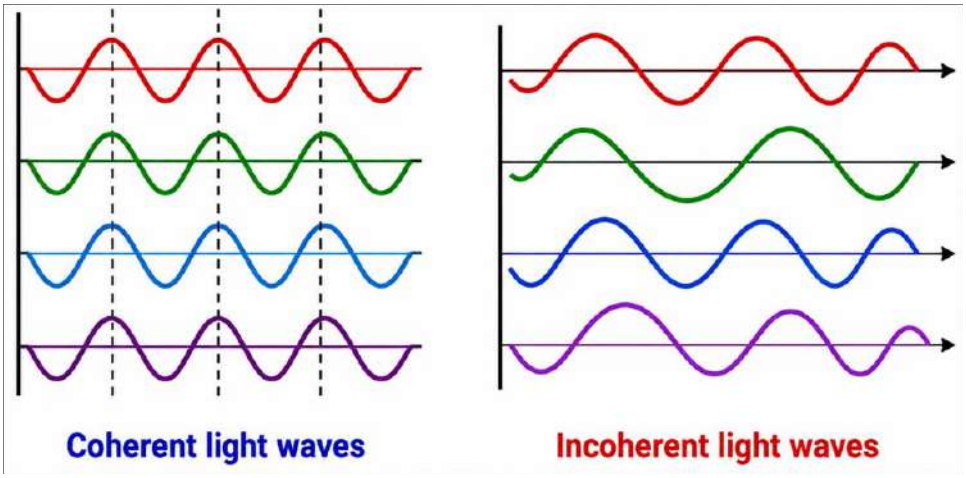


Figure 6: Coherence of laser light

of laser beam is concentrated in a small region. This concentration of energy gives a high intensity. Since in Laser many number of photons are in phase with each other, the amplitude of the resulting wave becomes na and hence the intensity of laser is proportional to n^2a^2. It is estimated that light from a typical 1mW laser is 10,000 times brighter than the light from the sun at the earth's surface.

4.3.2 Difference between laser light and ordinary light-

S. No.	Ordinary Light	Laser Light
1	Light emitted is not monochromatic	Light emitted is highly monochromatic
2	Light is not intense and bright	Light is intense and bright
3	Light is emitted in all directions	Light is emitted only in one direction
4	Light does not have a high degree of coherence	Light has a high degree of coherence
5	Coherence time is not higher	Coherence time is much higher

4.3.3 Difference between Stimulated and Spontaneous Emission:

S. No.	Stimulated Emission	Spontaneous Emission
1	An atom in the excited state is induced to return to the ground state, thereby resulting in two photons of the same frequency and energy. This process is called stimulated emission.	The atom in the excited state returns to the ground state by emitting a photon without any external inducement. This process is called spontaneous emission.
2	The emitted photons move in the same direction and are highly directional.	The emitted photons move in all directions and are random.
3	The radiation is highly intense, monochromatic, and coherent.	The radiation is less intense and is incoherent.
4	The photons are in phase.	The photons are not in phase.
5	The rate of transition is given by: $R_{21}(St) = B_{21}u_\nu N_2$	The rate of transition is given by: $R_{21}(Sp) = A_{21}N_2$

4.3.4 Necessary condition to achieve laser action:

1. **The rate of emission must be greater than the rate of absorption-** Under normal condition, the number of atoms in the upper energy level is always smaller than that in lower energy level. If by some means, the number of atoms in higher energy state is made greater than that in lower energy state, the emission rate will become greater than the absorption rate.

2. **The probability of spontaneous emission must be negligible in comparison to the probability of stimulated emission-** This condition can be achieved by taking working substance(active medium) such that its atom have metastable states which have a life time 10^{-3} sec. or more instead of usual 10^{-8} sec. If certain atoms are excited to metastable state the probability of spontaneous emission will be quite negligible.

3. **The probability of spontaneous emission must be negligible in comparison to the probability of stimulated emission-**This condition can be achieved by taking working substance(active medium) such that its atom have metastable states which have a life time 10^{-3} sec. or more instead of usual 10^{-8}sec. If certain atoms are excited to metastable state the probability of spontaneous emission will be quite negligible.

Population Inversion: Let N_1 be the number of atoms in ground state and N_2 be the number of atoms in excited state. For the laser action to take place, the higher energy levels should be more populated than the lower energy levels, i.e., $N_2 > N_1$. The situation in which the number of atoms in the higher energy state exceeds that in the lower state ($N_2 > N_1$) is known as "population inversion".

Light amplification: Let us consider many numbers of atoms in the excited state. We know the photons emitted during stimulated emission have same frequency, energy and are in phase as the incident photon. Thus results in 2 photons of similar properties. These two photons induce stimulated emission of 2 atoms in excited state thereby resulting in 4 photons. These 4 photons induce 4 more atoms and give rise to 8 photons etc.

Due to stimulated emission the photons multiply in each step giving rise to an intense beam of photons that are coherent and moving in the same direction. Hence the Light is Amplified by Stimulated Emission of Radiation, termed as LASER.

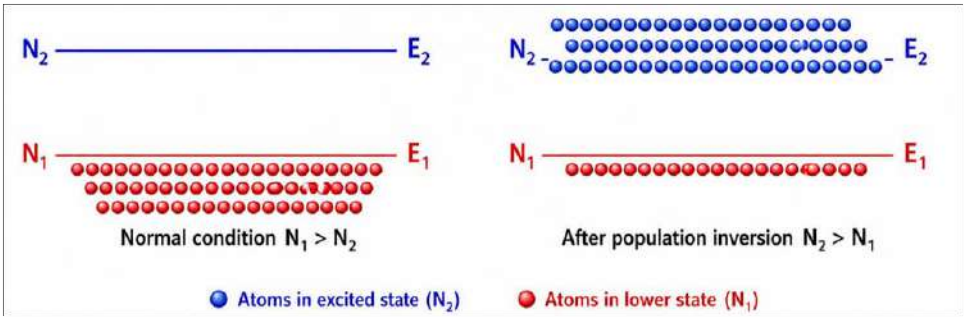


Figure 7: Population Inversion

4.3.5 Pumping methods:

The process of achieving population inversion is called pumping. Pumping can be classified into the following types based on the type of source of pumping.

1. **Optical pumping:**Here the atoms are excited with the help of photons emitted by an optical source. The atoms absorb energy from the photons and raises to excited state. Optical pumping is used in solid laser. Examples : Ruby Laser, Nd-YAG Laser

2. **Electrical pumping:**Electrical discharge pumping is used in gas lasers. Since gas lasers have very narrow absorption. The electrons are accelerated to very high velocities by strong electric field and they collide with gas atoms and these atoms are raised to excited state. Examples : Argon Laser, CO2 Laser, He-Ne Laser

3. **Chemical pumping:** Due to some chemical reactions, the atoms may be raised to excited state. Examples : Dye Laser.

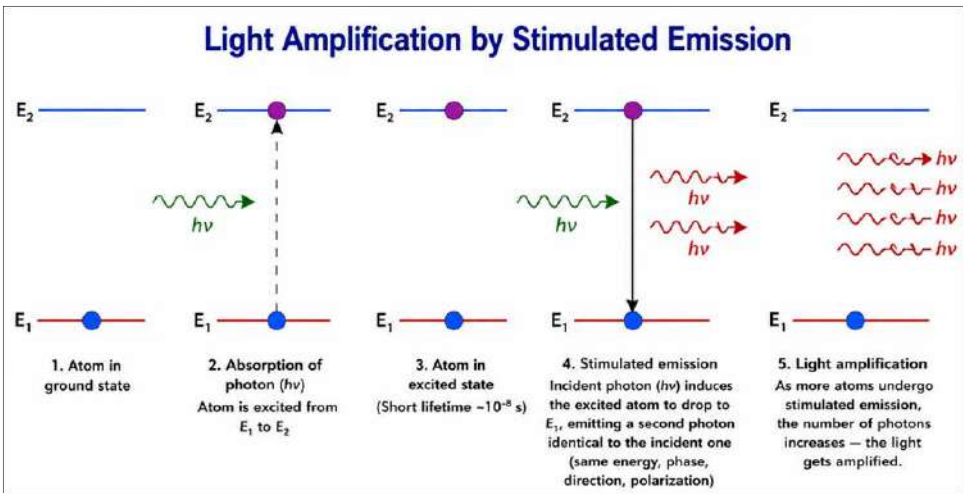


Figure 8: Schematic of light amplification

4.3.6 Metastable state:

It is an excited state of an atom with a longer life time than the other excited states. Atoms in the metastable state remain excited for a considerable time in the order of 10^{-6} to 10^{-3} seconds. Such relatively long-lived states are called as Metastable state. An atom can exist in a metastable energy level for a longer time before radiating than itcan in an ordinary energy level. An atom can be excited to a higher level by supplying energy to it. Normally, excited atoms

have short life times and release their energy in a matter of 10^{-8} seconds through spontaneous emission. It means atoms do not stay long to be stimulated. As a result, they undergo spontaneous emission and rapidly return to the ground level; thereby population inversion could not be established. In order to do so, the excited atoms are required to 'wait' at the upper energy level till a large number of atoms accumulate at that level, that is A Meta stable state. These levels lie in the forbidden gap of the host crystal. There could be no population inversion and hence no laser action, if metastable states don't exist.

4.4 Components of Lasers

Active Medium: It is the material in which the laser action takes place. The active medium may be solid crystals such as liquid dyes, gases like CO2 or Helium-Neon, or semiconductors such as GaAs. This medium decides the wavelength of laser radiation. Active mediums contain atoms which can produce more stimulated emission than spontaneous emission and cause amplification they are called "Active Centers".

Pumping Energy Source (Excitation Mechanism): Energy Source (Excitation mechanisms) pumps the active centers from ground state to excitedstate to achieve population inversion. The pumping by energy source can be optical, electricalor chemical depending on the active medium.

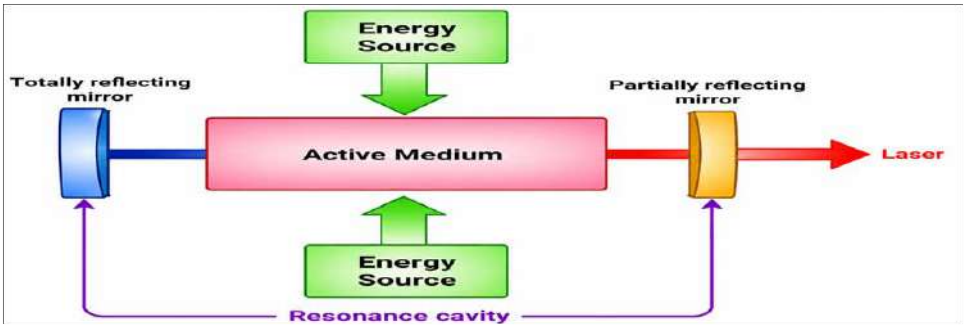


Figure 9: Optical Resonator

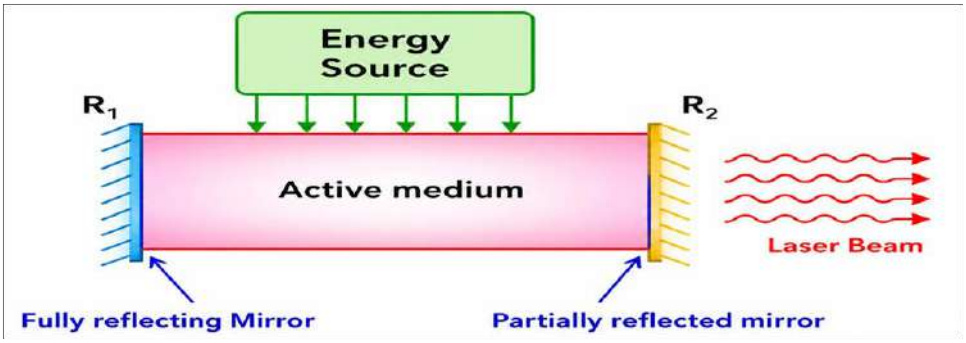


Figure 10: Optical Resonator

Optical Resonator: The optical resonator consists of two reflecting mirrors R_1 and R_2. The mirror R_1 is fully reflecting while the other mirror R_2 is partially reflecting. The active material is placed between them. The photon generated due to transitions between the energy states of active material undergoes multiple reflections between the two mirrors. As the photons (light) bounces back and forth between the two mirrors, the intensity of light is increased enormously. Finally the intense and amplified beam (laser) is allowed to come out the partial

mirror R_2. Therefore, the function of the optical resonator is to increase the intensity of laser beam.

4.5 Types of Laser

4.5.1 Ruby Laser (Solid-State Laser)

A Ruby laser is the first successful laser, developed by Theodore H. Maiman in 1960. It is a three-level solid-state laser in which the active medium is a synthetic ruby crystal consisting of Al_2O_3 (aluminium oxide) doped with about 0.05% Cr^{3+} (chromium) ions.

Construction: The ruby laser consists of a ruby rod. which is made of chromium doped ruby material. At the opposite ends of this rod there are two silver polished mirrors. Whose one is fully polished andother is partially polished. A spring is attached to the rod with fully polished end for adjustment of wave length of the laser light. Around the ruby rod a flash light is kept for the pump input. The whole assembly is kept in the glass tube. Around the neck of the glass tube the R.F source and switching control is designed in order to switch on and off the flash light for desired intervals.

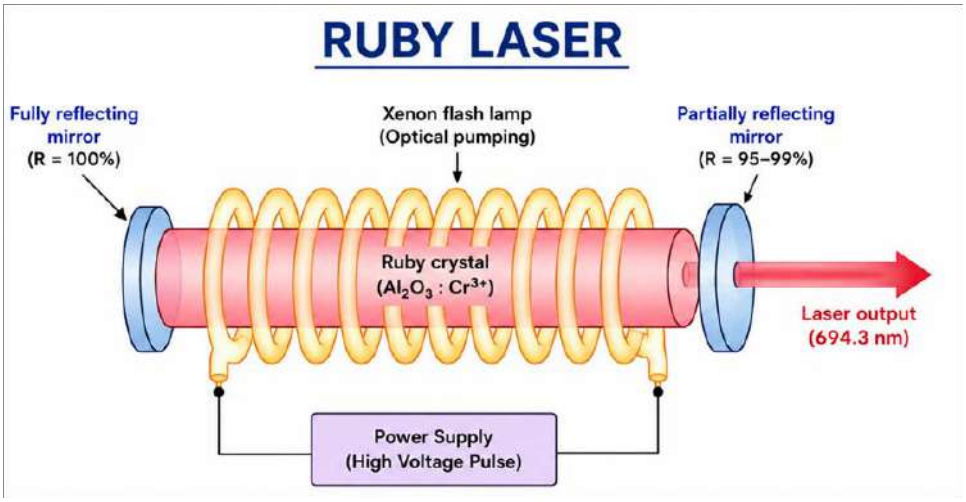


Figure 11: Ruby laser.

Operation of Ruby Laser: When we switch on the circuit the R.F operates. As a result, the flash of light is obtained aroundthe ruby rod this flash causes the electrons within ruby rod to move from lower energy band towards higher. This flash causes the electrons within ruby rod to move from lower energy band towards higher energy band. The population inversion take place at high energy band and electrons starts back to travel towards the lower energy band. During this movement the electron emits the laser light. This emitted light travels between the two mirrors where cross reflection takes place of this light. The stimulated laser light now escapes from partially polished mirror in shape of laser beam. The switching control of the R.F source is used to switch on and off the flash light so that excessive heat should not be generated due to very high frequency of the movement of the electron.

Energy Level Diagram for Ruby Laser The above three level energy diagram show that in ruby lasers the absorption occurs, this makes raise the electrons from ground state E_1 to the band of level E_3 higher than E_1. At E_3 these excited levels are highly unstable and so the electrons decays rapidly to the level of E_2. This transition occurs with energy difference (E_1–E_2) given up as heat (radiation less transmission). The level E_2 is very important for stimulated emission process and is known as Meta stable state. Electrons in this level have an average life time ofabout 5ms before theyfall to ground state. After this the population inversion can

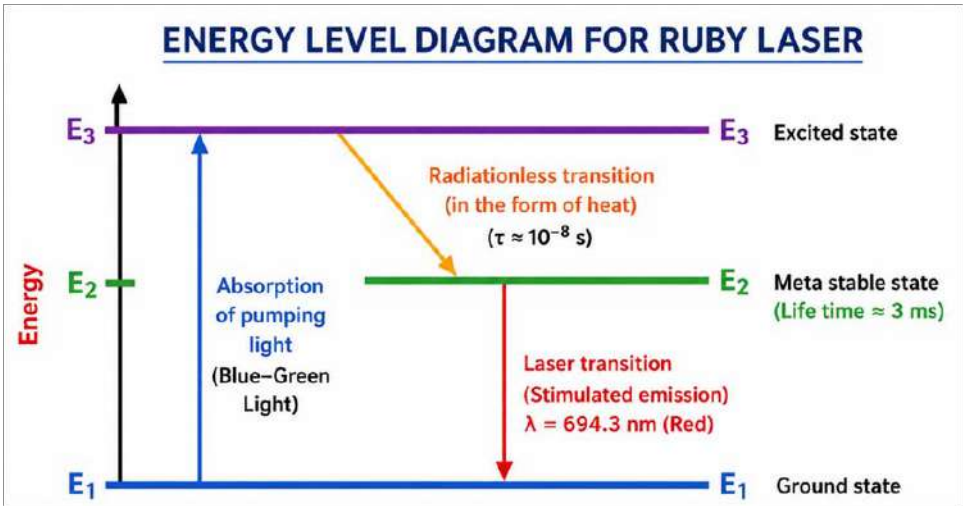


Figure 12: Ruby laser: Energy levels.

be established between E_2 and E_1. The population inversion is obtained by opticalpumping of the ruby rod with a flash lamp. When the flash lamp intensity becomes large enough to create population inversion, then stimulated emission from the Meta stable level to the ground level occurs which result in the laser output.

Advantages of Ruby Lasers

- Beam diameter of the ruby laser is comparatively less than CO2 gas lasers.
- Output power of Ruby laser is not as less as in He-Ne gas lasers.
- Since the ruby is in solid form therefore there is no chance of wasting material of active medium.
- Construction and function of ruby laser is self-explanatory.

Disadvantages of Ruby Laser

- In ruby lasers no significant stimulated emission occurs, until at least half of the ground state.
- Electrons have been excited to the Meta stable state.
- Efficiency of ruby laser is comparatively low.
- Optical cavity of ruby laser is short as compared to other lasers, which may be considered a disadvantage.

Applications of Ruby Laser

- Due to low output power they are class-I lasers and so may used as toys for children.
- It can be used in schools, colleges, universities for science programs.
- It can be used as decoration piece & artistic display.

4.5.2 Helium -Neon (He-Ne) Laser

He-Ne Laser is the first gas laser fabricated and operated successfully by Ali Javan and his coworkers in Bell Telephone Laboratories in 1961. This employs electrical pumping and are capable of supply continuous laser beam. This laser is trouble free and have extremely long operating life time and is operates in the low-pressure mixture of He & Ne- gases.

Construction: Active medium: It is a gas laser, which consists of a narrow quartz tube filled with a mixture of Helium and Neon gases in the ratio 10:1 respectively, at low pressure (∼0.1 mm of Hg). Ne atoms act as active centers and responsible for the laser action, while He atoms are used to help in the excitation process. The length of the quartz tube is about 50 cm and the diameter is about 1 cm.

Optical resonator: To construct the optical resonator cavity, two parallel mirrors are placed at the ends of the quartz tube one of them is partly transparent while the other is fully reflecting. The spacing between the mirrors is adjusted such that it should be equal to the integral multiple of half- wavelengths of the laser light.

Pumping system: The pumping is done through electrical discharge by using electrodes that are connected to a high frequency alternating current source.

Principle and working: The common helium-neon gas laser achieves a population inversion in a different way. A mixture of about 10 parts of Helium and 1 part of Neon at a low pressure is placed in a glass tube that has parallel mirrors, one of them partly transparent, at both ends. The spacing of the mirrors is equal to an integral number of half-wavelengths of the Laser light. An electric discharge is produced in the gas by means of electrodes outside the tube connected to a source of high-frequency alternating current, and collisions with electrons from the discharge excite He and Ne atoms to metastable states respectively above their ground states. Some of the excited He atoms transfer their energy to ground-state Ne atoms in collisions. The purpose of the He atoms is thus to help achieve a population inversionin the Ne atoms.

The laser transition in Ne is from the metastable state to an excited state with the emission of a 650nm of photon. Then another photon is spontaneously emitted in a transition to a lower metastable state; this transition yields only incoherent light.

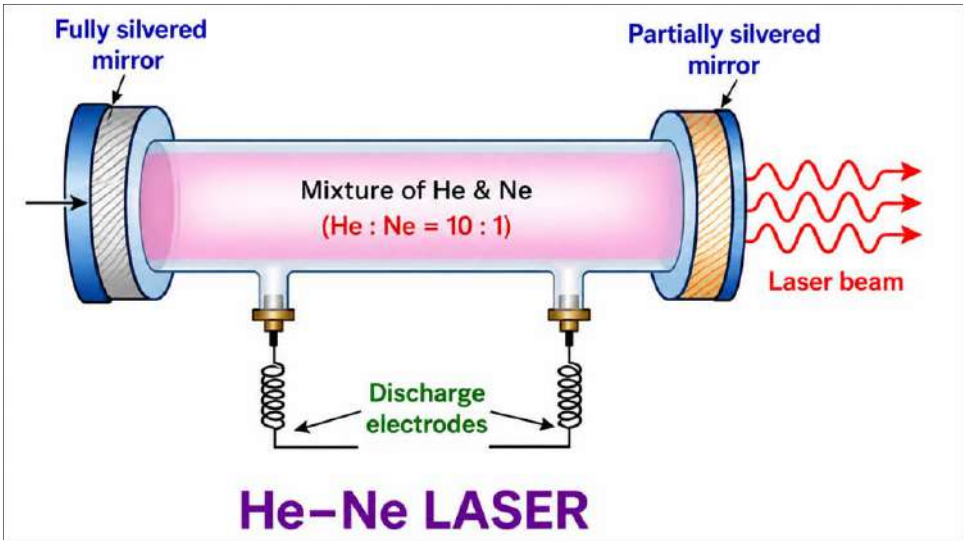


Figure 13: He-Ne laser.

Characteristics of He-Ne Laser The He-Ne laser is a relatively low power device with an output in the visible red portion of the spectrum. The most common wavelength produced by He-Ne lasers is 632.8nm. Majority of He-Ne lasers generate less than 10m watt of power, but some can be obtained commercially with up to 50m watts of power. For He-Ne lasers the

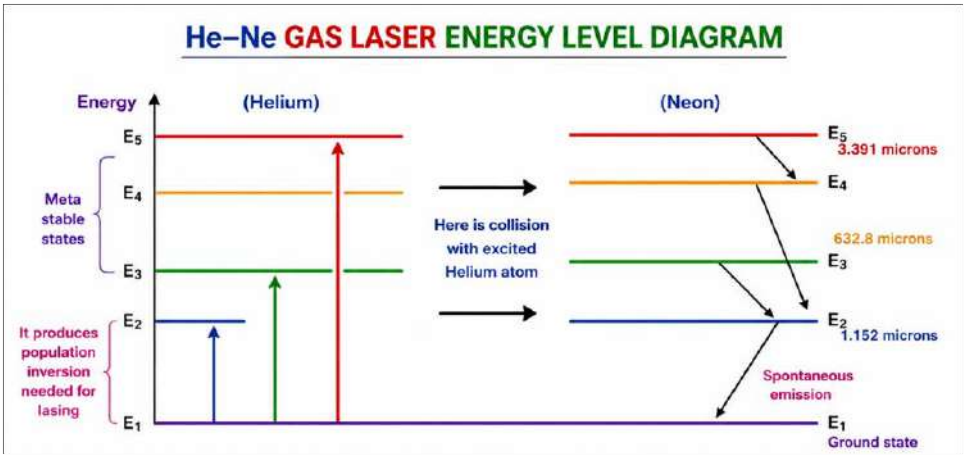


Figure 14: He-Ne laser: Energy levels.

typical laser tube is from 10 to 100 cm in length and the life time of such a tube can be as high as 20,000 hours.

Applications of He-Ne Laser The Helium-Neon gas laser is one of the most commonly used Laser today because of the following applications.

- Many schools / colleges / universities use this type of laser in their science programs and experiments.
- He-Ne lasers also used in super market checkout counters to read bar codes and QR codes.
- The He-Ne lasers also used by newspapers for reproducing transmitted photographs.
- He-Ne lasers can be use as an alignment tool.
- It is also used in Guns for targeting.

Advantages of He-Ne Laser

- Ne laser has very good coherence property.
- He-Ne laser tube has very small length approximately from 10 to 100cm and best lifetime of 20.000 hours.
- Cost of He-Ne laser is less from most of other lasers.
- Construction of He-Ne laser is also not very complex.
- He-Ne laser provide inherent safety due to low power output.

Disadvantages of He-Ne Laser

- It is relatively low power device means its output power is low.
- He-Ne laser is low gain device.
- High voltage requirement can be considered its disadvantage.
- Escaping of gas from laser plasma tube is also its disadvantage.

4.5.3 Semiconductor laser

Semiconductor laser technology has been growing explosively for the past several years. It is based on the combination of optical and semiconductor electronic technology. The most detailed proposal for semiconductor lasers came from Nikolai Basov's group at Lebedev institute of physics, Moscow in 1961.

Main Features and physical consideration: The main features that distinguish the semiconductor laser from the other types laser are:

1. They have very small size (typically 300mm×10mm×50μm) which enables it to be incorporated easily into other instruments.

2. They can be directly pumped by low power electric current (typically 15Ma at 2volts) which makes it possible to drive it with conventional circuitry.

3. They have high efficiency of converting electrical electric power to laser light (typically ¿20 percent).

4. They have semiconductor-based manufacturing technology which lends itself to mass production.

5. Their output beams are compatible with those of typical silica based optical fibres and there is a possibility of tailoring their output wavelength to low-loss low-dispersion region in such fibres.

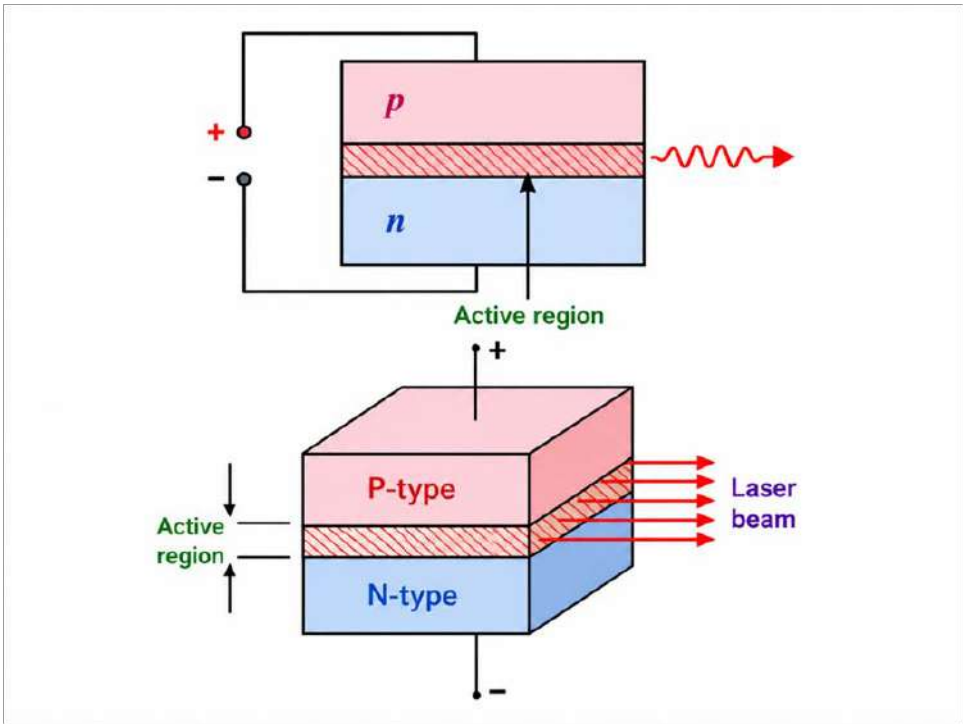


Figure 15: Semiconductor laser.

Principle: In semiconductormaterial when electron and hole recombine, excess of energy is either emitted as heat energy or light energy. In case of Ge or Si the emitted energy is in the form of heat, so of no use in laser action. Whereas in case of GaAs, CdS, CdSe, the energy is emitted in the form of light radiation near infrared region and can be of great importance in lasing. So the principle of laser is conversion of electrical energy to optical energy.

Laser Action in p-n junction: The most common way of producing population inversion in the semiconductor is by joining p-type and n-type material together (as shown in figure), the resulting arrangement is known as p-n junction. The first laser involving transition between the energy bands of GaAs p-n junction was reported by Hall and co-workers in 1962 who achieved direct conversion of electrical energy to coherent infrared radiation.

Construction and working: The diode is used in the form of cube with each side of a mm length and the junction lying in the horizontal plane through the centre of cube. The depletion layer is usually very thin $\sim$0.1 μm shown as shaded area in the figure. The front and back faces are polished parallel to each other and are perpendicular to the plane of junction, thus forming an optical cavity. The other two faces are left with a rough finish to suppress oscillations in unwanted directions. The active region is wider than the depletion region of thickness $\sim$1 μm.When forward bias is applied to p-n junction, electrons from n-side and holes from p-side will be injected into junction area.

The p-n junction will experience transition of electrons from the conduction band to the valence band, the electrons and holes recombine thereand thus emits extra energy as radiation.

As material is placed within suitable optical resonator, lasing action takes place. The laser output of GaAs oscillates at the wavelength 8500Å to 9000Å near infrared region.

4.5.4 Applications of Laser

Lasers are used extensively in industry, medicine, communication, scientific research, defense, and entertainment because they produce highly coherent, monochromatic, intense, and directional light.

- Cutting, welding, drilling, engraving, marking, surface hardening, additive manufacturing (3D printing).
- LASIK eye surgery, retinal surgery, dentistry, tattoo removal, kidney stone fragmentation (lithotripsy), cancer treatment (photodynamic therapy), cosmetic skin treatments.
- Optical fiber communication, high-speed internet, satellite communication, free-space optical communication.
- Spectroscopy, holography, interferometry, Raman spectroscopy, laser cooling, plasma research, nonlinear optics.
- Laser range finders, target designation, missile guidance, laser radar (LiDAR), directed-energy weapons, surveillance.
- CD/DVD/Blu-ray players, laser printers, barcode scanners, semiconductor chip fabrication (photolithography).
- LiDAR mapping, distance measurement, topographic surveys, atmospheric pollution monitoring.
- Laser light shows, stage lighting, projection displays, holographic displays.
- Precision length measurement, alignment of machines, calibration, interferometers.
- Satellite ranging, adaptive optics, space communication, lunar laser ranging.
- Fingerprint scanners, biometric systems, security alarms, anti-counterfeit holograms.
- Laser pointers, optical computer mouse, laser levels, projectors. Major Applications

4.6 Fiber Optics

Introduction: Construction and Basic Principle

Fiber optics, the science of transmitting data, voice, and images by the passage of light through thin, transparent fibers. In telecommunications, fiber optic technology has virtually replaced copper wire in long-distance telephone lines, and it is used to link computers within local area networks. Fiber optics is also the basis of the fiberscope's used in examining internal parts of the body (endoscopy) or inspecting the interiors of manufactured structural products. The basic medium of fiber optics is a hair-thin fiber that is sometimes made of plastic but most often of glass. A typical glass optical fiber has a diameter of 125 micrometers (μm). It consists of three regions—

(a) **CORE** - It is the innermost cylinder with diameter 10-100 μm and refractive index n_1. Light beam is kept within the core using the phenomenon of Total Internal reflection.

(b) **CLADDING** - The core is surrounded by a glass or plastic material with refractive index n_2. It provides some strength to core and keep light waves within the core. The difference between n_1 and n_2 is 10^{-3} μm.

(c) **SHEATH** - The core cladding system is enclosed in an outer jacket to protect it from abrasions, contaminations and moisture.

Many such fibers with individual jackets are grouped as cables that contain hundreds of fibers.

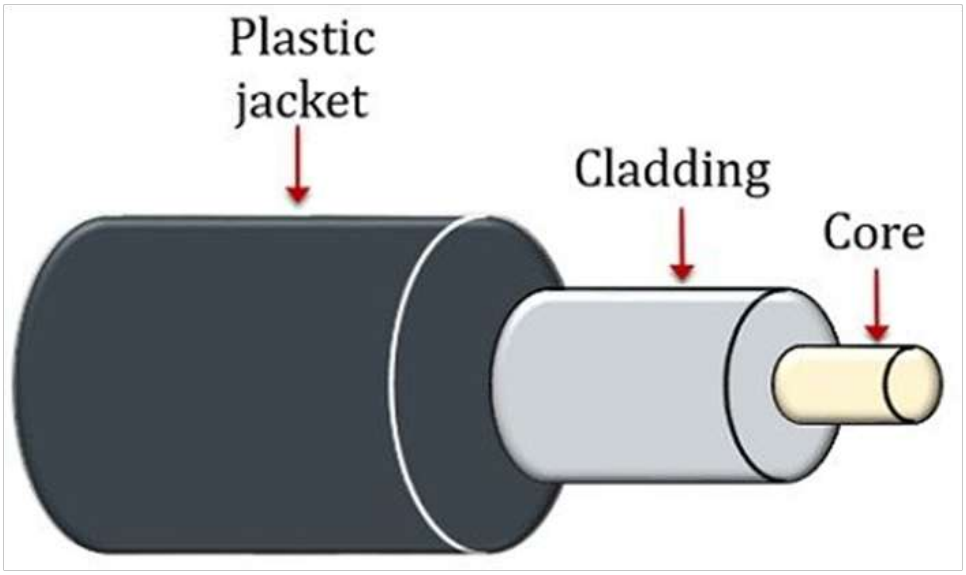


Figure 16: Construction of fiber.

Basic principle: Principle of Optical Fibre

The working principle of an optical fibre is based on the Total Internal Reflection (TIR).

- When light travels from a denser medium (core) to a rarer medium (cladding) at an angle greater than the critical angle, it gets completely reflected back into the core.
- This reflection happens repeatedly along the fibre, allowing light to travel long distances with very little loss.
- Because of TIR, the light signal remains confined within the fibre and does not escape outside.

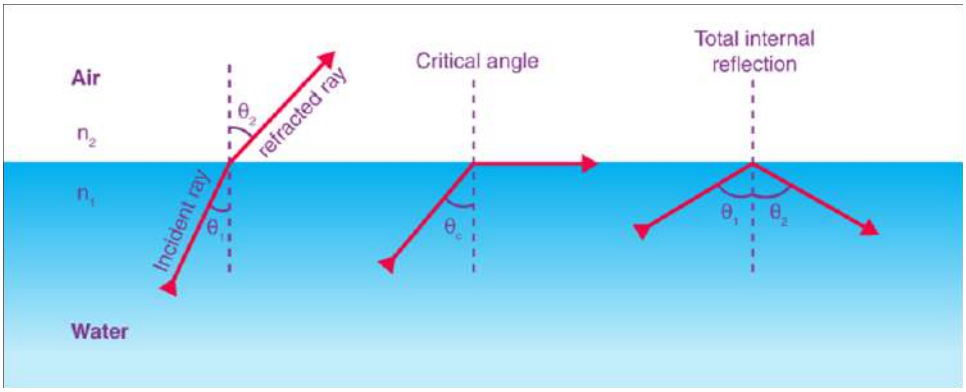


Figure 17: Total internal reflection.

4.6.1 TYPES OF FIBER-

There are three types of optical fiber to meet different requirements- (a) Step Index Multi mode Fiber (b) Step Index Single mode Fiber (c) Graded Index Multi mode Fiber

STEP INDEX MULTI MODE FIBER- A step index multimode fibre is an optical fibre in which the core has a uniform refractive index and there is a sudden (step-like) decrease in refractive index at the cladding boundary. Because the core (about 100μm large) is relatively large, multiple light rays (modes) can enter and propagate through the fibre. These rays travel in zig-zag paths, undergoing repeated Total Internal Reflection at the core–cladding interface. Due to different path lengths of these rays, the signal spreads out in time, causing modal dispersion, which limits bandwidth and makes this fibre suitable mainly for short-distance communication such as LAN systems. These fibers are used for short distances 200m range.

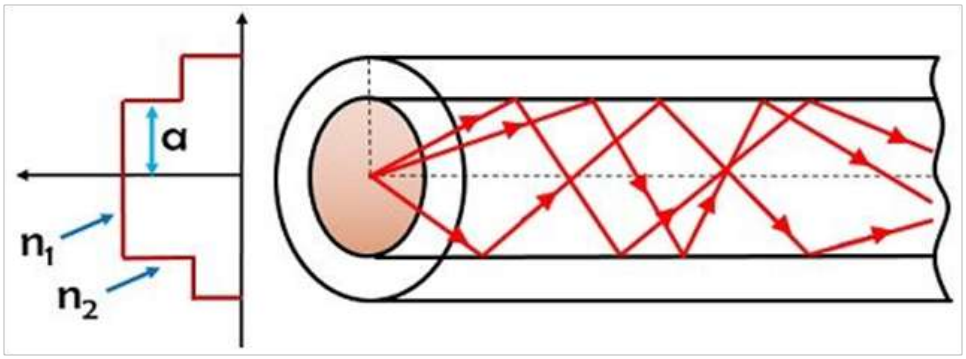


Figure 18: Step index multi-mode fibre.

STEP INDEX SINGLE MODE FIBER- A step index single mode fibre is an optical fibre in which the core has a uniform refractive index with a sharp (step) decrease at the cladding, and it allows only one light ray (mode) to propagate. The core is very thin (about 8–10 μm), so light travels almost in a straight path along the axis of the fibre with minimal reflections, following the principle of Total Internal Reflection. Because only one mode is present, there is no modal dispersion, resulting in high bandwidth and low signal loss, making it ideal for long-distance and high-speed communication systems such as optical networks and telecommunication cables.

GRADED INDEX MULTI MODE FIBER- A graded index multimode fibre is an optical fibre in which the refractive index of the core gradually decreases from the center towards the cladding, forming a smooth (parabolic) profile. Due to this variation, multiple light rays (modes) travel through the fibre in curved paths instead of sharp zig-zag reflections. Rays near the outer region move faster than those near the center, which helps them reach the output at

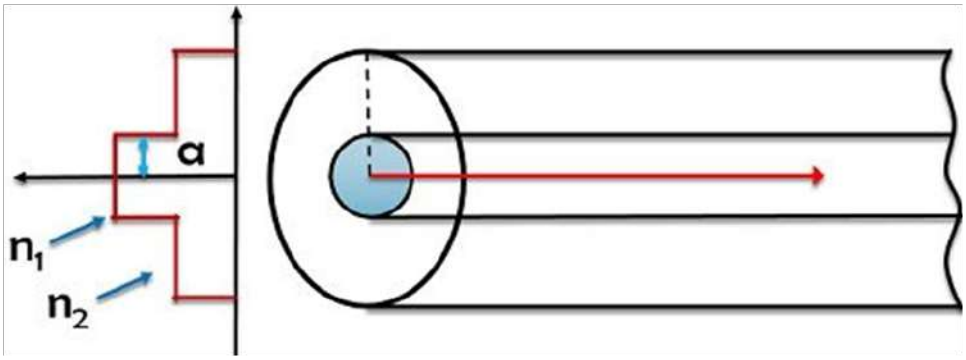


Figure 19: Step index single-mode fibre.

nearly the same time, thereby reducing modal dispersion. The fibre operates on the principle of Total Internal Reflection and provides higher bandwidth and better performance than step index multimode fibre, making it suitable for medium-distance communication systems.

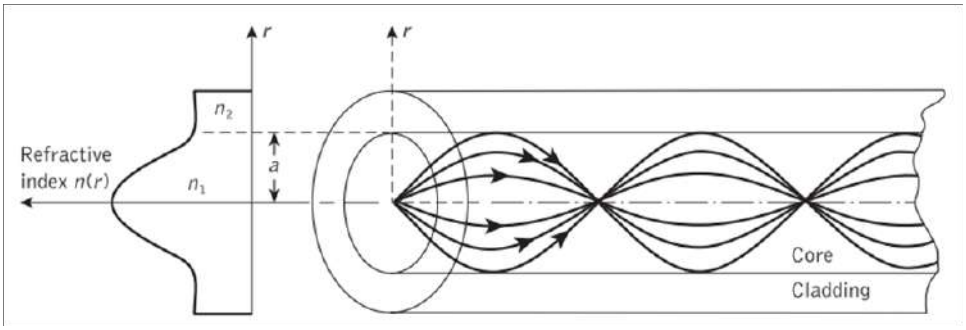


Figure 20: Graded index multi-mode fibre.

4.6.2 Difference in Step Index Single Mode Fibre and Step Index Multimode Fibre:

Feature	**Step Index Single Mode Fibre**	**Step Index Multimode Fibre**
Core Diameter	Very small (8–10 μm)	Large (50–100 μm)
Number of Modes	Only one mode	Multiple modes
Light Propagation	Straight (axial path)	Zig-zag (multiple reflections)
Refractive Index Profile	Step index (uniform core)	Step index (uniform core)
Dispersion	Negligible (no modal dispersion)	High modal dispersion
Bandwidth	Very high	Low
Transmission Distance	Long distance	Short distance
Signal Loss	Very low	Higher compared to single mode
Cost	Expensive	Cheaper
Applications	Long-distance telecom, internet	LAN, short-distance communication

4.6.3 Difference in Step Index and Graded Index Fibre:

Feature	Step Index Fibre	Graded Index Fibre
Refractive Index Profile	Sudden (step-like) change between core and cladding	Gradually decreases from center to edge
Light Path	Zig-zag (sharp reflections)	Smooth curved path
Core Structure	Uniform refractive index	Varying refractive index (parabolic)
Dispersion	High modal dispersion	Low modal dispersion
Bandwidth	Low	Higher than step index
Signal Quality	Lower	Better
Cost	Cheaper	Costlier
Applications	Short-distance communication	Medium-distance communication

4.6.4 Acceptance Angle and Numerical Aperture

Optical fibers are widely used for transmitting light signals over long distances with minimal loss. Two important parameters that describe the performance of an optical fiber are the acceptance angle and the numerical aperture (NA).

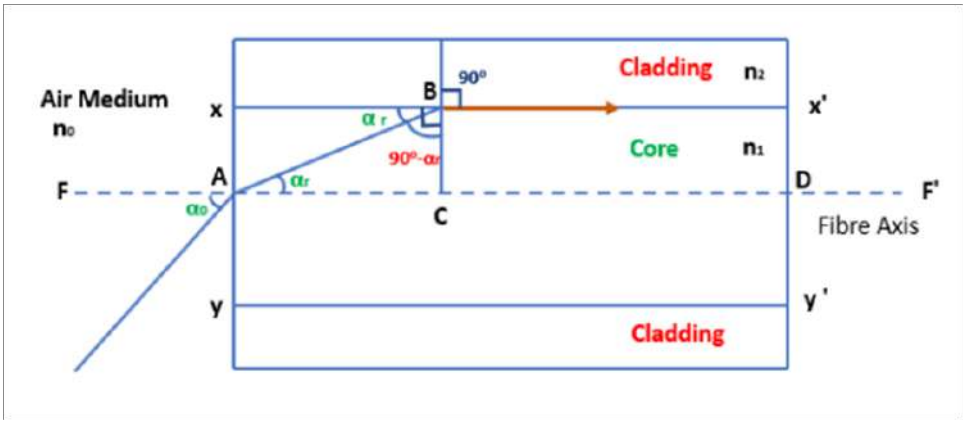


Figure 21: Numerical Aperture.

Let light enters at point A with angle α_0 (acceptance angle) and inside core, it refracts at angle α_r. Applying Snell's Law at point A:

$$n_0 \sin \alpha_0 = n_1 \sin \alpha_r \tag{1}$$

At point B, the ray strikes the core–cladding boundary and undergoes Total Internal Reflection. From the diagram:

$$\text{Angle between ray and normal} = 90^\circ - \alpha_r$$

For limiting condition (just total internal reflection):

$$i = C$$

So,

$$90^\circ - \alpha_r = C$$
$$\alpha_r = 90^\circ - C$$

Use Critical Angle Relation

$$\sin C = \frac{n_2}{n_1}$$

Now,

$$\sin \alpha_r = \sin(90^\circ - C) = \cos C$$
$$\cos C = \sqrt{1 - \sin^2 C} = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

Thus,

$$\sin \alpha_r = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

Substitute into Equation (1)

$$n_0 \sin \alpha_0 = n_1 \times \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$
$$n_0 \sin \alpha_0 = \sqrt{n_1^2 - n_2^2}$$

For air ($n_0 = 1$):

$$\sin \alpha_0 = \sqrt{n_1^2 - n_2^2}$$

The maximum angle of incidence (α_0) at point A, for which a light ray entering the optical fibre undergoes total internal reflection at point B, is called the acceptance angle.

Numerical Aperture (NA) of an optical fiber is defined as the light-gathering capacity of the fiber.

From the derivation of acceptance angle, we already have:

$$n_0 \sin \alpha_0 = \sqrt{n_1^2 - n_2^2}$$

Substituting this into the definition of NA:

$$NA = n_0 \sin \alpha_0$$
$$NA = \sqrt{n_1^2 - n_2^2}$$

For air $n_0 = 1$:

$$NA = \sin \alpha_0$$

Numerical aperture depends only on the refractive indices of the core and cladding and determines the amount of light that can be accepted and guided through the optical fiber.

Numerical Aperture in terms of Relative Refractive Index (Δ):

$$NA \approx n_1 \sqrt{2\Delta}$$

where,

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Applications of Optical Fiber

- **Communication:** Used in telephone, internet, and cable TV for high-speed transmission.
- **Medical Field:** Used in endoscopy and laser-based surgeries.
- **Industrial Use:** Fiber optic sensors measure temperature, pressure, and strain.
- **Defense & Security:** Provides secure communication and surveillance systems.
- **Networking & Research:** Used in LANs, data centers, and scientific experiments.